# MAT 137 (Calculus II) Prof. Swift

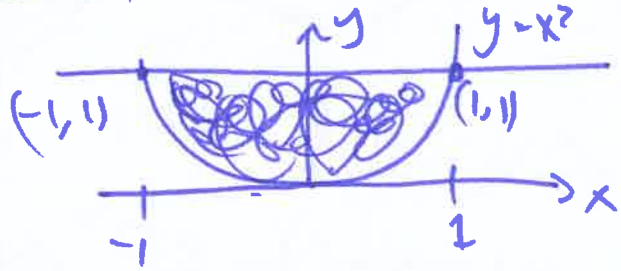
**In-class worksheet: Computing Volume of a Solid of Revolution**

Let $\mathcal{R}$ be the region in the $x$-$y$ plane between the curves $y = x^2$ and $y = 1$.

1. Find the volume of the solid obtained when $\mathcal{R}$ is rotated about the $x$-axis.

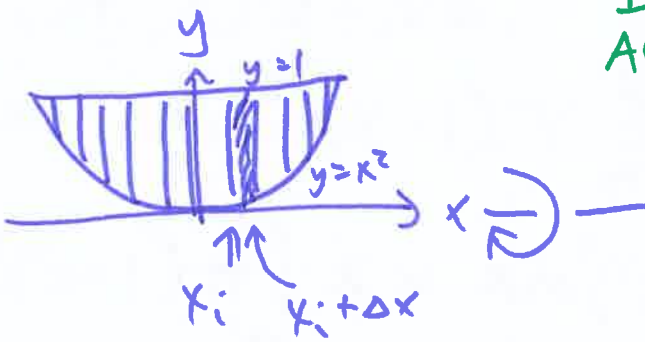
2. Find the volume of the solid obtained when $\mathcal{R}$ is rotated about the line $y = 1$.

First, sketch $\mathcal{R}$, including points of intersection.




Note: I would give almost full credit for this original solution. I wrote $r_{in} = x_i^2$ and $A(x_i) = \pi(r_{out}^2 - r_{in}^2)$. I just forgot to square ($x_i^2$). Show your work for partial credit

1.





$A(x_i) = \pi(r_{out}^2 - r_{in}^2)$ ← yes

$= \pi(1 - (x_i^2)^2)$ ← NO! Needs square!

$\Delta V_i = A(x_i)\Delta x = \pi(1 - x_i^{\cancel{2}\,4})\Delta x$

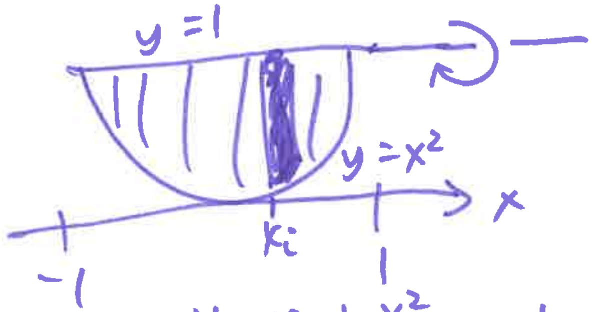
$V \approx \sum_{i=0}^{n-1} \Delta V_i$, so $V = \int_{-1}^{1} \pi(1 - x^{\cancel{2}\,4})\,dx = 2\int_0^1 \pi(1 - x^{\cancel{2}\,4})\,dx$

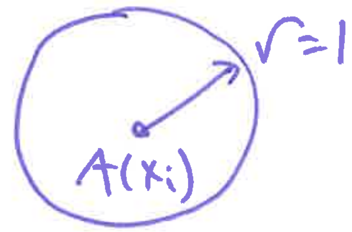
since the integrand, $f(x) = \pi(1 - x^{\cancel{2}\,4})$ is even.

So $V = 2\pi \int_0^1 (1 - x^{\cancel{2}\,4})\,dx = 2\pi\left(x - \frac{x^{\cancel{3}\,5}}{\cancel{3}\,5}\right)\Big|_0^1 = 2\pi\left(1 - \frac{1}{\cancel{3}\,5}\right) - 0$

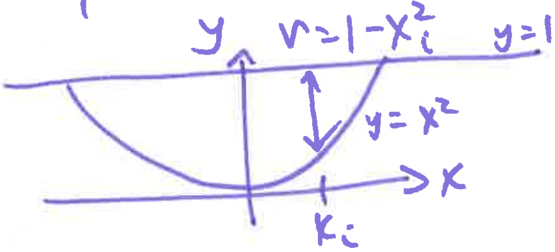
$\boxed{\cancel{V = 2\pi \cdot \frac{2}{3} = \frac{4\pi}{3}}}$ $\boxed{V = 2\pi \cdot \frac{4}{5} = \frac{8\pi}{5}}$

2. Now, rotate $R$ about the line $y=1$.




solid is a pointy football.




$$\Delta V_i = \left(\pi r^2\right) \Delta x$$

$$\Delta V_i = \pi \left(1 - x_i^2\right)^2 \Delta x$$

$$V \approx \sum_{i=0}^{n-1} \Delta V_i = \sum_{i=0}^{n-1} \pi \left(1 - x_i^2\right)^2 \Delta x$$ . Take limit $n \to \infty$, $\Delta x \to 0$.

$$V = \int_{-1}^{1} \pi \left(1 - x^2\right)^2 dx = 2 \int_0^1 \pi \left(1 - x^2\right)^2 dx$$

compute the integral.

$$V = 2\pi \int_0^1 \left(1 - x^2\right)^2 dx = 2\pi \int_0^1 \left(1 - 2x^2 + x^4\right) dx$$

$$= 2\pi \left(x - \frac{2x^3}{3} + \frac{x^5}{5}\right)\Big|_0^1 = 2\pi \left(1 - \frac{2}{3} + \frac{1}{5}\right) - 0$$

$$V = 2\pi \, \frac{15 - 10 + 3}{15} = 2\pi \cdot \frac{8}{15}$$

$$\boxed{V = \frac{16}{15}\pi}$$