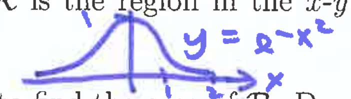
MAT 137 (Calculus II) Prof. Swift

Quiz 3, on Applications of the Integral. Name: key

For this quiz, you *may* work with other people. You may not use calculators or any internet-connected device. You may leave the class after you turn in your quiz.

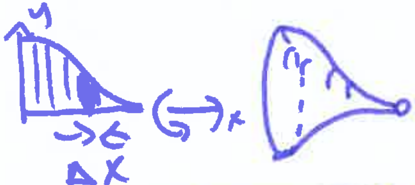
In each of these problems, $\mathcal{R}$ is the region in the x-y plane bounded by $y = e^{-x^2}$, $y = 0$, $x = 0$ and $x = 2$.



1. Set up a definite integral to find the area of $\mathcal{R}$. Do not evaluate the integral.

$$\boxed{A = \int_0^2 e^{-x^2}\,dx}$$ ← This is NOT elementary. It cannot be evaluated with pencil and paper.

2. Set up a definite integral to find the volume of the solid obtained when $\mathcal{R}$ is rotated about the x-axis. Do not evaluate the integral.

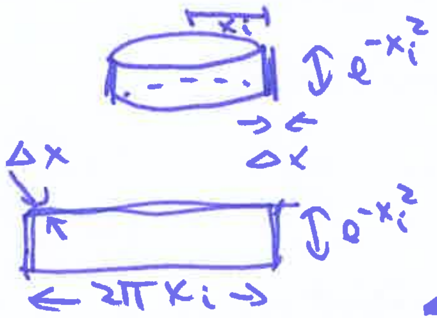


Let ΔV_i be the bit of volume of the disk swept out by the rectangle between $x = x_i$ and $x = x_i + \Delta x$.

$$\Delta V_i = \pi\left(e^{-x_i^2}\right)^2 \Delta x = \pi e^{-2x_i^2}\Delta x$$

So $$\boxed{V = \int_0^2 \pi e^{-2x^2}\,dx} \quad \left(= \lim_{n\to\infty} \sum_{i=0}^{n-1} \pi e^{-2x_i^2}\Delta x\right) \quad x_0 = 0,\ x_n = 2$$

3. Set up a definite integral to find the volume of the solid obtained when $\mathcal{R}$ is rotated about the y-axis. This time, *do* evaluate the integral.



Let ΔV_i be the volume of the cylindrical shell obtained when the rectangle between $x = x_i$ and $x = x_i + \Delta x$ is swept around the y-axis.

$$\Delta V_i = 2\pi x_i e^{-x_i^2} \Delta x$$

$$\text{so } V = \lim_{n \to \infty} \sum_{i=0}^{n-1} 2\pi x_i e^{-x_i^2} \Delta x \qquad (x_0 = 0,\ x_n = 2)$$

$$\boxed{V = \int_0^2 2\pi x e^{-x^2}\, dx} = \pi \int_0^4 e^{-u}\, du = -\pi e^{-u}\Big|_0^4$$

$$= -\pi e^{-4} + \pi e^0$$

$u = x^2$ $\quad x=2 \Rightarrow u=4$

$du = 2x\,dx$ $\quad x=0 \Rightarrow u=0$

$$\boxed{V = \pi(1 - e^{-4})}$$

3. Set up a definite integral to find the volume of the solid obtained when $\mathcal{R}$ is rotated about the y-axis. This time, *do* evaluate the integral.

If you really want to use pancakes, you can. But it isn't easy! There are 2 parts: $\underbrace{0 \le y \le e^{-4}}_{V_1}$, and $\underbrace{e^{-4} \le y \le 1}_{V_2}$.

$V = V_1 + V_2$.

$V_1 = \pi(2^2)e^{-4} = 4\pi e^{-4} =$ Volume of cylinder at bottom: 

1  $x_r = \sqrt{-\ln(y)}$ $A(y) = \pi(x_r)^2 = \pi(-\ln(y)) = -\pi\ln(y)$

$$V_2 = -\int_{e^{-4}}^{1} \pi \ln(y)\,dy = +\pi(y - y\ln(y))\Big|_{e^{-4}}^{1}$$

use integration by parts!

$$= \pi(1 - 5e^{-4})$$

The total volume is $V = V_1 + V_2 = 4\pi e^{-4} + \pi(1 - 5e^{-4})$

$$\boxed{V = \pi(1 - e^{-4})}$$