# MAT 137 (Calculus II) Prof. Swift

## In-class worksheet: Computing Volume by Cylindrical Shells

Let $f(x) = x + x^3$, and let $\mathcal{R}$ be the region in the $x$-$y$ plane bounded by the curves $y = f(x)$, $y = 0$, and $x = 1$.

1. Make a rough sketch of the graph $y = f(x)$. Hint: Calc 1 will help to make the sketch. Note that $f'(x) = 1 + 3x^2 > 0$ for all $x$.


$y = x + x^3$, or $y = f(x)$

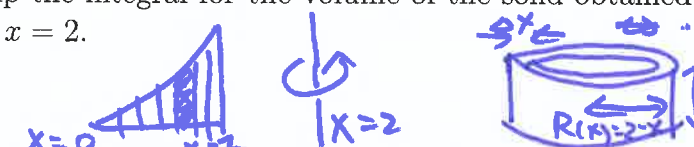
$f$ is odd, and increasing.

2. Sketch the region $\mathcal{R}$. Find the coordinates of the corners of the region.


3. Set up the integral for the volume of the solid obtained when $\mathcal{R}$ is rotated about the line $x = 2$.

$$\boxed{V = \int_0^1 2\pi(2-x)(x+x^3)\,dx}$$

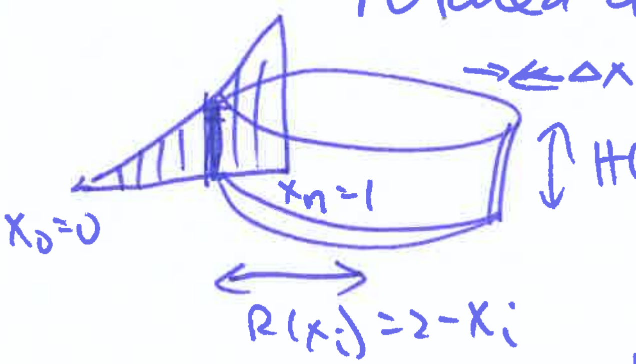
Shortcut:




4. What goes wrong if you try to compute the volume of that solid using washers?

Long answer to 3:

$\Delta V_i$ = volume ~~between~~ of the cylindrical shell obtained when the rectangle between $x = x_i$ and $x = x_i + \Delta x$ is rotated about $x=2$




$$\Delta V_i = 2\pi(2-x_i)(x_i + x_i^3)\Delta x$$

So $V = \lim_{n\to\infty} \sum_{i=0}^{n-1} 2\pi(2-x_i)(x_i+x_i^3)\Delta x$

you don't need to write this sum, but you do need to understand that this is happening behind the scenes.

So
$$\boxed{V = \int_0^1 2\pi(2-x)(x+x^3)\,dx}$$

4.


$y = x + x^3$

$x = ???$ as fn. of $y$

$V = \int_0^2 \pi\left((r_o(y))^2 - (r_i(y))^2\right)dy$

But we can't find $r_o(y)$!