MAT 137 (Calculus II) Prof. Swift

In-class worksheet: Arc Length

1. Set up the integral for the length of the curve $y = \cos(x)$ with $0 \le x \le \pi$.

Recall that $\frac{d}{dx}\sinh(x) = \cosh(x)$, $\frac{d}{dx}\cosh(x) = \sinh(x)$, and $\cosh^2(x) - \sinh^2(x) = 1$.

2. Set up the integral for the length of the curve $y = \cosh(x)$ with $0 \le x \le 1$.

3. Set up the integral for the length of the curve $y = 2\cosh(x)$ with $0 \le x \le 1$.

4. Frequently the integrals that compute arc length are not elementary. Only the integral in problem 2 is elementary. Evaluate it. Hint: Simplify the integrand.

1. $y = \cos(x)$, $\frac{dy}{dx} = -\sin(x)$, $L = \int_0^\pi \sqrt{1 + (-\sin(x))^2}\,dx$

$\boxed{L = \int_0^\pi \sqrt{1+\sin^2(x)}\,dx}$ NOT elementary.

2. $y = \cosh(x)$, $\frac{dy}{dx} = \sinh(x)$, $L = \int_0^1 \sqrt{1 + (\sinh(x))^2}\,dx$

$\boxed{L = \int_0^1 \sqrt{1+\sinh^2(x)}\,dx}$. See problem 4.

3. $y = 2\cosh(x)$, $\frac{dy}{dx} = 2\sinh(x)$, so

$\boxed{L = \int_0^1 \sqrt{1+4\sinh^2(x)}\,dx}$, not elementary

4. The identity $\cosh^2(x) - \sinh^2(x) = 1$ means
$1 + \sinh^2(x) = \cosh^2(x)$, so

$L = \int_0^1 \sqrt{\cosh^2(x)}\,dx = \int_0^1 \cosh(x)\,dx = \sinh(x)\Big|_0^1 = \sinh(1) - \underbrace{\sinh(0)}_{0}$

since $\cosh(x) > 0$

since $\frac{d}{dx}\sinh(x) = \cosh(x)$

$\boxed{L = \sinh(1)}$

or $\boxed{L = \frac{e - e^{-1}}{2}}$