MAT 137 (Calculus II) Prof. Swift

In-class worksheet: Work

Isaac Newton discovered that the force of gravity on an object at or above the surface of the earth is inversely proportional to the square of the distance from the center of the earth to the object.

1. How much work (in mile-pounds) does it take to lift a 200 pound boulder from the surface of the earth to a geosynchronous orbit? The radius of the earth is 4,000 miles, and a geosynchronous orbit is at an altitude of 22,000 miles. (Thus, the orbit is 26,000 miles from the center of the earth.)

Hint: Let F be the gravitational force in pounds on the boulder, and let r be the distance from the center of the earth to the boulder, in miles. Isaac Newton discovered that $F = \frac{k}{r^2}$ for $r \geq 4{,}000$. Start by finding k, the constant of proportionality. You do not need to know the mass of the earth or the mass of the boulder or the gravitational constant G that you might have learned about in physics.

2. How much work does it take to lift that same boulder from the surface of the earth to infinity? (That is, how much work is done in propelling the boulder out of the earth's gravitational field?)

1. $F(r) = \frac{k}{r^2}$, so $F(4000) = \frac{k}{(4{,}000)^2} = 200$ ← boulder weighs 200 lb on surface of the earth

$\therefore\ k = 200 \cdot (4{,}000)^2 = 32 \times 10^8 = 3.2 \times 10^9$

so $F(r) = \frac{3.2 \times 10^9}{r^2}$

$$W = \int_{4000}^{26{,}000} \frac{3.2 \times 10^9}{r^2}\,dr = 3.2 \times 10^9 \int_{4{,}000}^{26{,}000} r^{-2}\,dr$$

$$W = 3.2 \times 10^9 \left(\frac{-1}{r}\right)\Big|_{4{,}000}^{26{,}000} = \boxed{3.2 \times 10^9 \left(\frac{-1}{26{,}000} + \frac{1}{4{,}000}\right)}$$

2. $$W = \int_{4{,}000}^{\infty} \frac{3.2 \times 10^9}{r^2}\,dr = \lim_{b \to \infty} \left[3.2 \times 10^9 \left(\frac{-1}{b} + \frac{1}{4{,}000}\right)\right] = \boxed{\frac{3.2 \times 10^9}{4{,}000}}$$

A nicer form of the answer is obtained by using original expression for k.

$$W = \frac{k}{4{,}000} = \frac{200 \cdot (4{,}000)^2}{4{,}000} = \underbrace{200}_{\text{weight at surface}} \cdot \underbrace{4{,}000}_{\text{radius of earth}} = \boxed{8 \cdot 10^5}$$