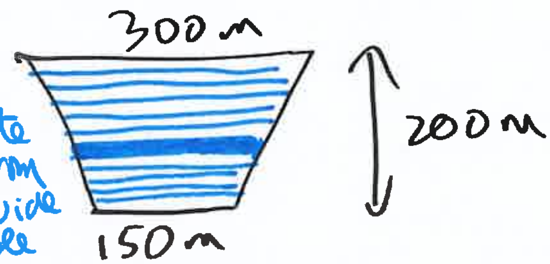
MAT 137 (Calculus II) Prof. Swift

In-class worksheet: Hydrostatic pressure

Here is a picture of Glen Canyon Dam, along with an idealized sketch.



Compute force from short, wide rectangle

Assume that the shape of the dam is a trapezoid, 200 meters tall, 150 meters wide at the bottom, and 300 meters wide at the top. Set up the integral for the force of the water on the dam, in newtons. Assume the dam is full, like it was in June of 1983 when it almost overflowed.

Hints: The pressure of water at depth h is $P(h) = \rho g h$. When pressure is constant, then Force = Pressure times Area. Your integral should have "ρg" as a factor. You do not need to evaluate the integral, and you do not need to replace ρg with 9.8×10^3 newtons per cubic meter.

You have two choices for variables. One choice is to do an integral with respect to h, where $h = 0$ at the top, and $h = 200$ at the bottom. Or, do an integral with respect to y, where $y = 0$ at the bottom and $y = 200$ at the top. In either case, divide the dam into many short, wide rectangles with height Δh or Δy.

I'll do it both ways: Start with y:

Let y = height, in meters, above bottom of the dam.

Let ΔF_i = Force on the dam due to rectangle at $y = y_i$, with height Δy

$= P(y_i) \cdot$ Area of strip

$= \rho g \underbrace{(200 - y_i)}_{\text{height below rim} = 200 - y_i} \cdot w(y_i)\, \Delta y$

what is width as a function of y?

$w(y)$ = width

$w(0) = 150$

$w(200) = 300$

w is linear, with slope

$\frac{300-150}{200-0} = \frac{150}{200} = \frac{3}{4}$

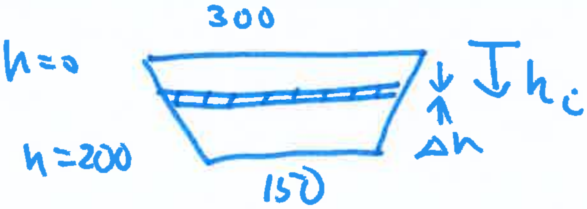
$w(y) = \frac{3}{4} y + 150$

$\Delta F_i = \rho g (200 - y_i)\left(\frac{3}{4} y_i + 150\right) \Delta y$

So

$$\boxed{F = \int_0^{200} \rho g (200 - y)\left(\frac{3}{4} y + 150\right) dy}$$

Now, do the calculation using h. This is more straight forward, but using y is what you want to do for problems 8 and 10.



Let ΔF_i = Force on the rectangle between $h=h_i$ and $h=h_i+\Delta h$

$= \text{Pressure} \cdot \text{Area}$

$= \rho g h_i \cdot w(h_i)\Delta h$

where $w(h)$ is the width of the dam at depth h.

$w(0)=300$, $w(200)=150$, so $w(h)=300-\frac{3}{4}h$.

$$\Delta F_i = \rho g h_i\left(300-\tfrac{3}{4}h_i\right)\Delta h$$

The total Force is $F \approx \sum_{i=0}^{n-1} \Delta F_i$

$$\boxed{F = \int_0^{200} \rho g h\left(300-\tfrac{3}{4}h\right)dh}$$

Bonus material!

Evaluating either the y-integral or the h-integral gives

$$F = \rho g\cdot(4\times 10^6) = 10^3\,\frac{kg}{m^3}\cdot 9.8\,\frac{m}{s^2}\cdot 4\cdot 10^6\,m^3$$

$F = 3.92\times 10^{10}\,N$ (1 Newton = 1 kg $\frac{m}{s^2}$.)

$F \approx 8.81\times 10^9$ lb (9.8 N $\cong$ 2.2 lb)

The Force is about 9 billion pounds

Note: 1kg weighs 9.8N, or about 2.2 pounds.