MAT 137 (Calculus II) Prof. Swift

Introduction to Differential Equations

1. Verify that $y = 3e^{x^2}$ is a solution to the Initial Value Problem $\frac{dy}{dx} = 2xy$, $y(0) = 3$.

$y(x) = 3e^{x^2}$

$y(0) = 3e^{0^2} \stackrel{?}{=} 3$

$3 \cdot 1 = 3$ ✓

$3e^{x^2} \cdot \frac{d}{dx}(x^2) \stackrel{?}{=} 2x(3e^{x^2})$

$3e^{x^2} \cdot 2x = 2 \cdot x \cdot 3e^{x^2}$ ✓

2. Verify that the function $y = c_1 e^x + c_2 e^{-x}$ is a solution to the Ordinary Differential Equation (ODE) $\frac{d^2y}{dx^2} = y$, also written as $y'' = y$, for any values of the constants c_1 and c_2. Find the solution to the ODE that satisfies the initial conditions $y(0) = 0$, $y'(0) = 2$.

$y = c_1 e^x + c_2 e^{-x}$

$y' = c_1 e^x - c_2 e^{-x}$

$y'' = c_1 e^x + c_2 e^{-x}$

Note: Same function of x.

$y''(x) = y(x)$ for all x.

I.C. $y(0) = c_1 e^0 + c_2 e^{-0} = c_1 + c_2 = 0$

$y'(0) = c_1 e^0 - c_2 e^{-0} = c_1 - c_2 = 2$

$\boxed{\begin{array}{l} c_1 + c_2 = 0 \\ c_1 - c_2 = 2 \end{array}}$ solve this.

Add $2c_1 + c_2 - c_2 = 0 + 2$

$2c_1 = +2$

$c_1 = +1$

$c_1 + c_2 = 0$

$1 + c_2 = 0$

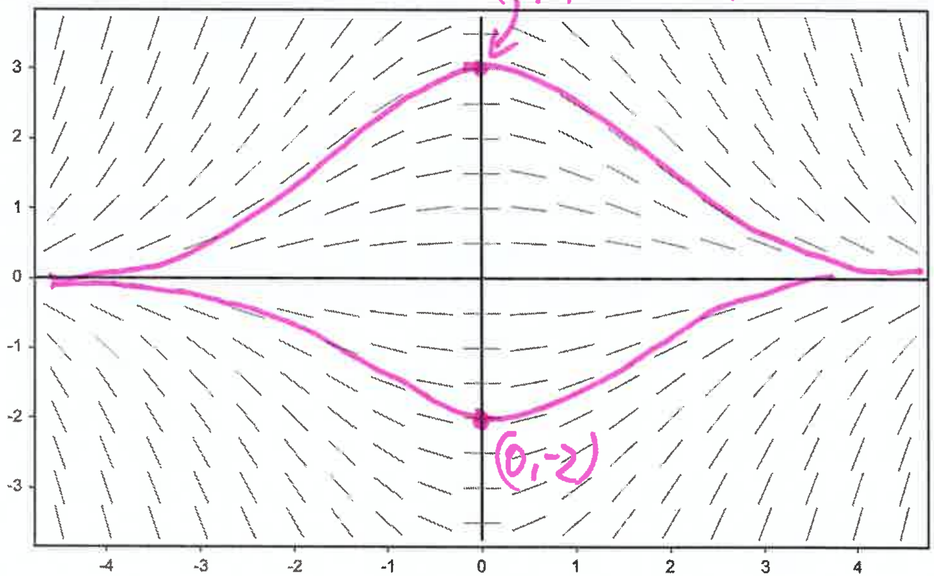
$c_2 = -1$

The solution that satisfies the IC's is $\boxed{y = e^x - e^{-x}}$ $(= 2\sinh(x))$

3. The following figure is the slope field for some ODE $y' = f(x, y)$. Sketch two solutions to the ODE: One that satisfies the initial condition $y(0) = 3$, and another that satisfies the initial condition $y(0) = -2$.

x=0, y=3
pt. (0,3)

(0,3)



y(0)=-2
x=0, y=-2
pt. (0,-2)