MAT 137 (Calculus II) Prof. Swift

Worksheet on Separation of Variables

1. Use separation of variables to find the solution to $\frac{dy}{dx} = y - 1$, $y(0) = 2$.

"Cross multiply" to get $\frac{dy}{y-1} = dx$

Integrate both sides: $\int \frac{dy}{y-1} = \int dx$

$\ln|y-1| + C_1 = x + C_2$

or $\ln|y-1| = x + (C_2 - C_1) = x + C$

Method A: Plug in $y(0) = 2$ now.

$\ln|2-1| = 0 + C$

$\ln|1| = C$; $C = 0$

$\ln|y-1| = x + 0$

$|y-1| = e^x$

$y - 1 = \pm e^x$, $y = 1 \pm e^x$

choose $+$ so $y(0) = 2$

$y = 1 + e^x$

Method B: Solve for y first

$|y-1| = e^{x+c} = e^x \cdot e^c$

$y - 1 = (\pm e^c) e^x = \tilde{C} e^x$

$y = 1 + \tilde{C} e^x$. Find $\tilde{C} = 1$. $y = 1 + e^x$

2. Use separation of variables to find the solution to $\frac{dy}{dx} = 2xy^2$, $y(0) = 1$.

"Cross multiply" $\frac{dy}{y^2} = 2x\,dx$

Integrate: $\int y^{-2}\,dy = \int 2x\,dx$

$$\frac{y^{-1}}{-1} + C_1 = x^2 + C_2$$

$$\frac{-1}{y} = x^2 + (C_2 - C_1) = x^2 + C$$

I'll use method B:

$$y = \frac{-1}{x^2 + C}$$. Plug in $y(0) = 1$ ($x = 0 \Rightarrow y = 1$)

$$1 = \frac{-1}{0^2 + C} = \frac{-1}{C} \quad \therefore C = -1.$$

$$y = \frac{-1}{x^2 - 1} = \frac{1}{1 - x^2}$$

either form is OK.

Common mistake:

$-\frac{1}{y} = x^2$ (Leaving out C now is bad!!)

$y = \frac{-1}{x^2 + C}$ ← wrong!