MAT 137 (Calculus II) Prof. Swift

Worksheet on the Interval of Existence

1. Use separation of variables to find the solution to the initial value problem $\frac{dy}{dt} = y^2$, $y(0) = 1$. Note that t, not x, is the independent variable. This ODE is sometimes called the "explosion equation".

2. Find the interval of existence of the solution, and sketch the solution. You might want to use the slope field app on your phone or tablet to help. Note that the union of two intervals, like $(-\infty, 0) \cup (0, \infty)$, is *not* an interval.

3. For an arbitrary $y_0 > 0$, solve the initial value problem $\frac{dy}{dt} = y^2$, $y(0) = y_0$.

4. Find the interval of existence of the solution in problem 3. Note that the interval depends on y_0.

1. $\frac{dy}{dt} = y^2$, so $\frac{dy}{y^2} = dt$, so $\int y^{-2}\,dy = \int dt$

Integrate: $\frac{y^{-1}}{-1} = t + c$

$-\frac{1}{y} = t + C$

Plug in $t=0, y=1$ to satisfy $y(0)=1$

$-\frac{1}{1} + 0 + C \therefore C = -1$

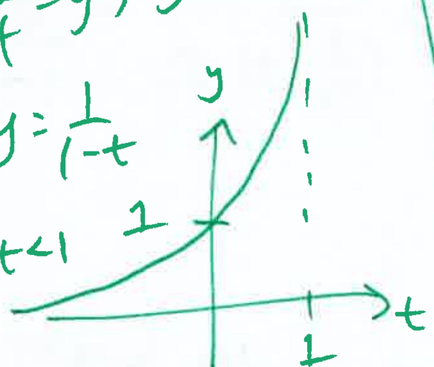
Plug $C=-1$ back into solution

$-\frac{1}{y} = t - 1$

$\frac{1}{y} = 1 - t$

$\boxed{y = \frac{1}{1-t}}$

Solution to $\frac{dy}{dt} = y^2$, $y(0)=1$ is $y = \frac{1}{1-t}$ for $-\infty < t < 1$



2. ~~the solution function is defined for $t \neq 1$. The~~ Note that $\frac{1}{1-t}$ is defined for $t \neq 1$. The largest interval containing $t=0$ on which the solution is defined is $-\infty < t < 1$.

The interval of existence is $(-\infty, 1)$.

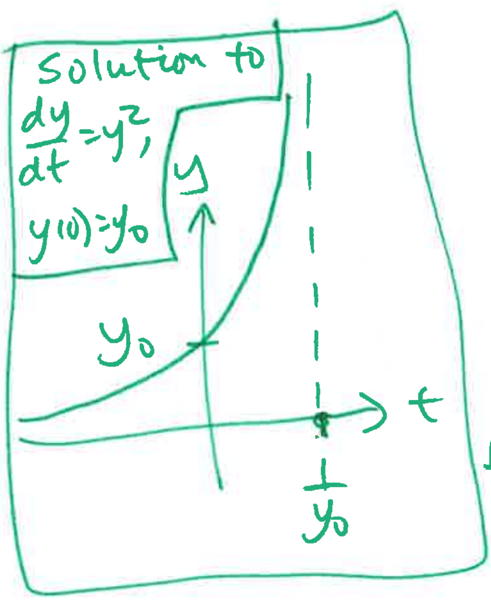
3. Now solve $\frac{dy}{dt} = y^2$, $y(0) = y_0$, for an arbitrary $y_0 > 0$.

We ~~get to~~ $\frac{-1}{y} = t + c$ as before.

Plug in $t=0$, $y = y_0$ & solve for c.

$$\frac{-1}{y_0} = 0 + c \quad \therefore \quad c = \frac{-1}{y_0}$$

Plug this c back into the solution & solve for y



$$\frac{-1}{y} = t - \frac{1}{y_0}$$

$$\frac{1}{y} = \frac{1}{y_0} - t$$

$$\boxed{y = \frac{1}{\frac{1}{y_0} - t}} = \frac{(1)\cdot y_0}{\left(\frac{1}{y_0} - t\right)\cdot y_0} = \frac{y_0}{1 - y_0 t}$$

both are OK

$$\boxed{y = \frac{y_0}{1 - y_0 t}}$$

4. The interval of existence is the largest interval containing $t=0$ on which $\frac{y_0}{1 - y_0 t}$ is defined. Set $1 - y_0 t = 0$. Solve for t: $y_0 t = 1$, $t = \frac{1}{y_0}$.

So the "blow up" time is $t = \frac{1}{y_0}$. The interval of existence is $-\infty < t < \frac{1}{y_0}$, or $\left(-\infty, \frac{1}{y_0}\right)$.