# MAT 137 (Calculus II) Prof. Swift

Worksheet on Modeling with First Order ODEs: Solutions

Suppose you drop a plush toy out of the window of a tall building. Let $v(t)$ be the speed of the toy, in feet per second, after $t$ seconds of free fall. Assume that the frictional force is proportional to the speed, so $v$ satisfies the ODE

$$\frac{dv}{dt} = 32 - kv,$$

where $k$ is a positive constant. You know that the terminal speed (usually called "terminal velocity") of the toy is 160 feet per second.

1. Find the value of $k$.

Since $v(t) = 160$ is a constant solution (which has $\frac{dv}{dt} = 0$) we know that $32 - k \cdot 160 = 0$. Solving for $k$ we find $k = 32/160$, or $\boxed{k = 1/5}$.

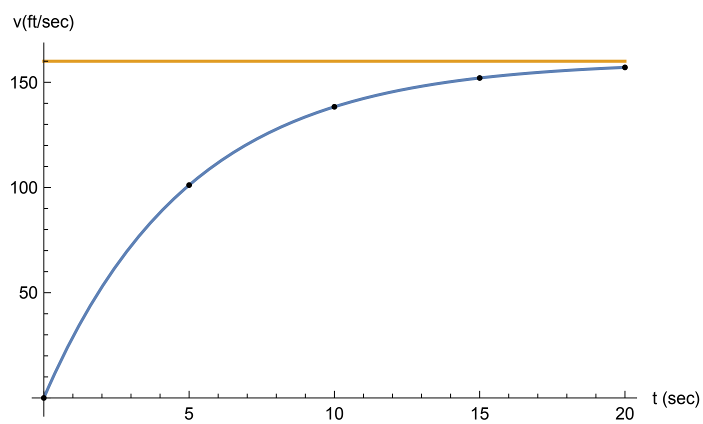
2. What is the initial speed $v(0)$?

$\boxed{v(0) = 0}$, since the plush toy is dropped.

3. Find the velocity $v(t)$ as an explicit function of $t$.

Using $k = 1/5$, the ODE is $\frac{dv}{dt} = 32 - \frac{1}{5}v = \frac{1}{5}(160 - v) = -\frac{1}{5}(v - 160)$. Separating the variables gives $\frac{dv}{v-160} = -\frac{1}{5}dt$. This form, with the unseen coefficient of 1 in front of $v$, is the best. Integrate both sides: $\ln|v - 160| = -t/5 + c$. Exponentiate both sides to get $|v - 160| = e^{-t/5} \cdot e^c$. Thus $v - 160 = \pm e^c e^{-t/5}$. While $e^c$ is always positive, we know that one solution to the ODE is $v = 160$. Thus the general solution to the ODE is $v(t) = 160 + Ce^{-t/5}$, where $C$ is any constant; positive, negative, or 0. Plug in $v(0) = 0$ to get $0 = 160 + Ce^0$, so $C = -160$. The solution to the IVP is

$$\boxed{v(t) = 160 - 160e^{-t/5}, \text{ or } v = 160(1 - e^{-t/5}).}$$




The speed $v(t)$ of the plush toy approaches the terminal speed of 160 feet per second exponentially, since $v(t) - 160 = Ce^{-t/5}$. Every 5 seconds, the difference between the speed of the plush toy and the terminal speed reduces by a factor of $e^{-1} \approx 1/3$.