MAT 137 (Calculus II) Prof. Swift

Worksheet on Newton's Law of Cooling

Newton's Law of Cooling says that the rate of change of the temperature of an object is proportional to the temperature difference between the object and its surroundings. This is also a "law of warming" for cold objects.

Suppose a cup of cold water, at 40° Fahrenheit, is placed in a 70° room at $t = 0$. Let $T(t)$ be the temperature of the water, in degrees Fahrenheit, after t minutes.

1. Write down the Initial Value Problem (IVP) for the temperature of the water, with an unknown positive constant of proportionality, k.

2. Solve the IVP using the trick discussed in class, or using separation of variables. You will get a function $T(t)$ that depends on the constant k.

3. After 10 minutes, the temperature of the water is 50 degrees. Find an expression for e^{-k}.

4. Use this expression for e^{-k} to get a relatively simple formula for $T(t)$.
Hint: $(x^2)^3 = x^{2\cdot 3}$.

5. When is the temperature 60 degrees?

1. $\frac{dT}{dt} = -k(T-70)$, $T(0) = 40$

2. $\frac{d}{dt}(T-70) = -k(T-70)$, so $T-70 = Ce^{-kt}$

or $T = 70 + Ce^{-kt}$. $T(0) = 40$, so $40 = 70 + Ce^0$

$C = -30$

$\boxed{T = 70 - 30e^{-kt}}$

3. $50 = 70 - 30e^{-k\cdot 10}$, so $-20 = -30e^{-k\cdot 10}$, $\frac{2}{3} = e^{-k\cdot 10}$,

$\boxed{e^{-k} = \left(\frac{2}{3}\right)^{\frac{1}{10}}}$

4. $T = 70 - 30(e^{-k})^t = 70 - 30\left(\left(\frac{2}{3}\right)^{\frac{1}{10}}\right)^t$, $\boxed{T = 70 - 30\cdot\left(\frac{2}{3}\right)^{\frac{t}{10}}}$

5. Solve for t: $60 = 70 - 30\cdot\left(\frac{2}{3}\right)^{\frac{t}{10}}$, $-10 = -30\cdot\left(\frac{2}{3}\right)^{\frac{t}{10}}$

$\frac{1}{3} = \left(\frac{2}{3}\right)^{\frac{t}{10}}$, $\ln\left(\frac{1}{3}\right) = \ln\left[\left(\frac{2}{3}\right)^{\frac{t}{10}}\right] = \frac{t}{10}\ln\left(\frac{2}{3}\right)$, so

$\boxed{t = 10\frac{\ln\left(\frac{1}{3}\right)}{\ln\left(\frac{2}{3}\right)}}$, or $\boxed{t = 10\frac{\ln\left(\frac{2}{3}\right)}{\ln(3)}}$ $\boxed{t = 10\frac{\ln\left(\frac{1}{3}\right)}{\ln\left(\frac{2}{3}\right)}}$