MAT 137 (Calculus II) Prof. Swift

Second Worksheet on Sequences

1. Suppose that $\lim_{n\to\infty} |a_n| = 0$. Use the squeeze theorem (also called the sandwich theorem) to prove that $\lim_{n\to\infty} a_n = 0$.

2. Find a formula for $f(n) = 1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-3) \cdot (2n-1) \cdot (2n+1)$, involving factorials.

1. $-|a_n| \le a_n \le |a_n|$ for all n.

$\lim_{n\to\infty} |a_n| = 0$ is given

same limit

So $\lim_{n\to\infty} (-|a_n|) = -\left(\lim_{n\to\infty} |a_n|\right) = -(0) = 0$.

∴ same limit

By the squeeze theorem, $\lim_{n\to\infty} a_n = 0$

2. $f(n) = 1\cdot 3 \cdot 5 \cdot \ldots \cdot (2n-3)(2n-1)(2n+1)$

$$= \frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot \ldots (2n-1)(2n)(2n+1)}{2\cdot 4\cdot \ldots (2n)}$$

$$= \frac{(2n+1)!}{(2\cdot 1)(2\cdot 2)\ldots(2n)}$$

$$= \frac{(2n+1)!}{(2\cdot 2\cdot 2\ldots\cdot 2)(1\cdot 2\cdot 3\ldots n)}$$

$$\boxed{f(n) = \frac{(2n+1)!}{2^n \cdot n!}}$$

check a few cases: $\cancel{f(5) = 1\cdot 3\cdot 5 \cdot 7 \cdot 9 \cdot 11}$ Too large!

$f(1) = 1\cdot 3 \stackrel{?}{=} \frac{3!}{2^1\cdot 1!} = \frac{3\cdot 2\cdot 1}{2} = 3\cdot 1$ ✓

$f(2) = 1\cdot 3\cdot 5 \stackrel{?}{=} \frac{1\cdot 2\cdot 3\cdot 4\cdot 5}{2^2\cdot 1\cdot 2!} = \frac{1\cdot \cancel{2}\cdot 3\cdot \cancel{4}\cdot 5}{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}} = 1\cdot 3\cdot 5$ ✓