MAT 137 (Calculus II) Prof. Swift

Worksheet on Finite and Infinite Geometric Series

1. Let $s_9 = \sum_{n=0}^{9} \left(\frac{2}{3}\right)^n$. Write out s_9, using "dot dot dot". Compute $s_9 - \frac{2}{3}s_9$, canceling as many terms as possible, and simplify the expression. Solve for s_9.

2. Use a similar technique to find and simplify $s_n = \sum_{i=0}^{n} \left(\frac{2}{3}\right)^i$.

3. Sum the series (i.e. the infinite series) by completing the sentence. Fill in your answer to part 2 for s_n, and then evaluate the limit, using common sense.

$$\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n = \lim_{n\to\infty} s_n = \lim_{n\to\infty} 3 - 3\left(\frac{2}{3}\right)^{n+1} = 3$$

1. $S_9 = \left(\frac{2}{3}\right)^0 + \left(\frac{2}{3}\right)^1 + \left(\frac{2}{3}\right)^2 + \dots + \left(\frac{2}{3}\right)^9$

$\frac{2}{3} S_9 = \left(\frac{2}{3}\right)^1 + \left(\frac{2}{3}\right)^2 + \dots + \left(\frac{2}{3}\right)^9 + \left(\frac{2}{3}\right)^{10}$

so $S_9 - \frac{2}{3}S_9 = \left(\frac{2}{3}\right)^0 + 0 + 0 + \dots + \left(0 - \left(\frac{2}{3}\right)^{10}\right)$

$\frac{1}{3} S_9 = 1 - \left(\frac{2}{3}\right)^{10}$

$\boxed{S_9 = 3 - 3\left(\frac{2}{3}\right)^{10}}$

2. $S_n = 1 + \left(\frac{2}{3}\right)^1 + \left(\frac{2}{3}\right)^2 + \dots + \left(\frac{2}{3}\right)^n$

$\frac{2}{3} S_n = \left(\frac{2}{3}\right)^1 + \left(\frac{2}{3}\right)^2 + \dots + \left(\frac{2}{3}\right)^n + \left(\frac{2}{3}\right)^{n+1}$

$\frac{1}{3} S_n = 1 - \left(\frac{2}{3}\right)^{n+1}$

$\boxed{S_n = 3 - 3\left(\frac{2}{3}\right)^{n+1}}$

3. Fill in the blanks: Note $\lim_{n\to\infty} \left(\frac{2}{3}\right)^{n+1} = 0$

$\boxed{\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n = \lim_{n\to\infty} 3 - 3\left(\frac{2}{3}\right)^{n+1} = 3}$