MAT 137 (Calculus II) Prof. Swift

Worksheet on Geometric and Telescoping Series

1. Find the sum of the following geometric series using the result shown in class.

$9 - 3 + 1 - \frac{1}{3} + \frac{1}{9} - \cdots = \frac{9}{1-(-\frac{1}{3})} = \frac{9}{1+\frac{1}{3}} = \frac{9}{\frac{4}{3}} = 9 \cdot \frac{3}{4} = \boxed{\frac{27}{4}}$

$a=9,\ r=-\frac{1}{3}$

2. Find the sum of the finite geometric series using the result shown in class.

$1 + 2 + 4 + 8 + \cdots + 1024 = \frac{1-2048}{1-2} = \frac{1-2048}{-1} = 2048-1 = \boxed{2047}$ *

$a=1$ 1st missing term $= 2048$, $r=2$

Problems 3 and 4 concern the telescoping series $\sum_{n=0}^{\infty} e^n - e^{n+1}$.

3. Compute the n partial sum, $s_n = \sum_{i=0}^{n} e^i - e^{i+1} = e^0 - e^1 + e^1 - e^2 + e^2 - e^3 + \cdots + e^n - e^{n+1}$

$= e^0 - e^{n+1} = \boxed{1 - e^{n+1}}$

4. Does $\sum_{n=0}^{\infty} e^n - e^{n+1}$ converge? Give the sum, or explain why the series does not converge.

No. The series does not converge because the sequence of partial sums does not converge.

* Note: $2048-1 = 2^{11}-1$ is a Mersenne number, but $2047 = 23 \times 89$, so 2047 is NOT a Mersenne prime:

skip $2^{11}-1$.

$2^2-1,\ 2^3-1,\ 2^5-1,\ 2^7-1,\ 2^{13}-1,\ 2^{17}-1,\ 2^{19}-1$ are all prime! These are called Mersenne primes.

The largest known prime number, $2^{82,589,933}-1$, is a Mersenne prime.