MAT 137 (Calculus II) Prof. Swift

Worksheet on Geometric Series and the Test for Divergence

1. Evaluate. $1 - 2 + 4 - 8 + \ldots + (-2)^n =$ $\frac{1-(-2)^{n+1}}{1-(-2)} = \frac{1-(-2)^{n+1}}{3}$

2. Evaluate. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots =$ $\frac{1}{1-(-\frac{1}{2})} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$.

3. Suppose you know that $\lim_{n\to\infty} a_n = 0$. What can you conclude? Circle one.

The series $\sum a_n$ converges to 0.

The series $\sum a_n$ converges, but not necessarily to 0.

The series $\sum a_n$ diverges.

The series $\sum a_n$ might converge, and it might diverge.

4. Suppose you know that $\lim_{n\to\infty} a_n = 1$. What can you conclude? Circle one.

The series $\sum a_n$ converges to 1.

The series $\sum a_n$ converges, but not necessarily to 1.

The series $\sum a_n$ diverges.

The series $\sum a_n$ might converge, and it might diverge.

5. Evaluate $\lim_{n\to\infty} \frac{\sin(n)}{n}$, using the squeeze theorem. Does $\sum_{n=1}^{\infty} \frac{\sin(n)}{n}$ converge? maybe.

$-1 \le \sin(n) \le 1$

so $\frac{-1}{n} \le \frac{\sin(n)}{n} \le \frac{1}{n}$

$\lim_{n\to\infty} \left(\frac{-1}{n}\right) = \lim_{n\to\infty} \left(\frac{1}{n}\right) = 0$. So $\lim_{n\to\infty} \frac{\sin(n)}{n} = 0$.

The test for Divergence is inconclusive.