MAT 137 (Calculus II) Prof. Swift

Worksheet on the Integral Test

Use the integral test to determine if $\sum_{n=3}^{\infty} \frac{1}{n\ln(n)}$ converges.

$\sum_{n=3}^{\infty} \frac{1}{n\ln(n)}$ converges if and only if $\int_3^{\infty} \frac{1}{x\ln(x)}\,dx$ converges.

$$\int \frac{1}{x\ln(x)} = \int \frac{1}{\ln(x)} \cdot \frac{1}{x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|\ln(x)| + C$$

let $u = \ln(x)$, $du = \frac{1}{x}\,dx$

$$\text{So } \int_3^{\infty} \frac{1}{x\ln(x)}\,dx = \lim_{b\to\infty} \int_3^{b} \frac{1}{x\ln(x)}\,dx = \lim_{b\to\infty} \Big[\ln|\ln(x)|\Big]_3^b$$

$$= \lim_{b\to\infty} \Big[\ln(\ln(b)) - \ln(\ln(3))\Big] = \infty$$

Since $\ln(\ln(b))$ grows without bound as $b\to\infty$.

Therefore $\boxed{\sum_{n=3}^{\infty} \frac{1}{n\ln(n)} \text{ diverges to } \infty}$

That is, $\boxed{\sum_{n=3}^{\infty} \frac{1}{n\ln(n)} = \infty}$

Note: $\ln(\ln(3)) = 0.094\ldots$

So basically $S_n = \sum_{i=1}^{n} \frac{1}{i\ln(i)} \approx \ln(\ln(n))$

$$S_{10^6} \approx \ln(\ln(10^6)) \approx \ln(13.8) \approx 2.6$$

So, adding one million terms gets you to about 2.6

The partial sums diverge to ∞ very slowly.