MAT 137 (Calculus II) Prof. Swift

Worksheet on Absolute Convergence - Group Work

Indicate whether each series is Absolutely convergent (A), Convergent but not absolutely convergent (C), Divergent (D), or we don't have the tools to determine which (?).

1. C $\quad 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$ · Converges by the alternating series test
· $1 + \frac{1}{2} + \frac{1}{3} + \ldots$ diverges by the integral test.

2. ? $\quad \sum_{n=1}^{\infty} \frac{\sin(n)}{n}$

A $\quad \sum_{n=1}^{\infty} (-0.25)^n$ is geometric with $r = -0.25 = -\frac{1}{4}$, so the series converges
$\sum_{n=1}^{\infty} |(-0.25)^n| = \sum_{n=1}^{\infty} (0.25)^n$ is geo with $r = \frac{1}{4}$,
so $\sum (-0.25)^n$ converges absolutely

D $\quad \sum_{n=0}^{\infty} \frac{(-1)^n (n+1)}{n+2}$

$\lim_{n \to \infty} (-1)^n \frac{(n+1)}{n+2}$ DNE. So the series diverges by the test for divergence.

Footnotes:

1. $\boxed{\text{the alternating harmonic series converges, and the harmonic series diverges.}}$
$\boxed{\text{There for the alternating harmonic series converges conditionally.}}$
You may quote these facts on any exam, without proving the result.

2. $\sum_{n=1}^{\infty} \frac{\sin(n)}{n}$ is NOT alternating
Is $\sum_{n=1}^{\infty} \left| \frac{\sin(n)}{n} \right| = \sum_{n=1}^{\infty} \frac{|\sin(n)|}{n}$ convergent?
The comparison test is inconclusive, since
$\frac{|\sin(n)|}{n} \le \frac{1}{n}$, and $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.