MAT 137 (Calculus II) Prof. Swift

Worksheet on Geometric Series and Power Series

1. $\sum_{n=0}^{\infty} 4r^n = 4 + 4r + 4r^2 + \cdots$ converges to $\dfrac{4}{1-r}$ if $|r| \leq 1$ and diverges if $|r| \geq 1$.

2. $\sum_{n=0}^{\infty} 4x^n = 4 + 4x + 4x^2 + \cdots$ $(r = x)$ converges to $\dfrac{4}{1-x}$ if $|x| \leq 1$ and diverges if $|x| \geq 1$.

   The interval of convergence of the power series is all x such that $-1 < x < 1$, which is $(-1, 1)$ in interval notation. The radius of convergence is $R = 1$.

3. $\sum_{n=0}^{\infty} (2x)^n = 1 + 2x + 4x^2 + \cdots$ $(r = 2x)$ converges to $\dfrac{1}{1-2x}$ if $|x| < \frac{1}{2}$ and diverges if $|x| \geq \frac{1}{2}$.

   The interval of convergence of the power series is all x such that $-\frac{1}{2} < x < \frac{1}{2}$, which is $(-\frac{1}{2}, \frac{1}{2})$ in interval notation. The radius of convergence is $R = \frac{1}{2}$.

Series 3 converges if $|2x| < 1$ (or $|x| < \frac{1}{2}$)

and diverges if $|2x| \geq 1$ (or that is, $x < -\frac{1}{2}$ or $x > \frac{1}{2}$)