MAT 137 (Calculus II) Prof. Swift

Power Series Representations of Some Functions

Use the known geometric series $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$ to write out the first 4 nonzero terms in a power series representation of the given functions. Fill in the blank with the coefficients in $f(x) = \sum_{n=0}^{\infty} c_n x^n$. Find the radius of convergence of the power series.

1. $f(x) = \frac{1}{1+3x} = \frac{1}{1-(-3x)} = 1 + (-3x) + (-3x)^2 + (-3x)^3 + \ldots = 1 - 3x + 9x^2 - 27x^3 + \ldots$

$c_0 = 1$, $c_1 = -3$, $c_2 = 9$, $c_3 = -27$. The radius of convergence is $R = \frac{1}{3}$.

since the series converges iff $|-3x| < 1$. $|x| < \frac{1}{3}$

2. $f(x) = \frac{1}{1-x^2} = 1 + x^2 + (x^2)^2 + (x^2)^3 + \ldots = 1 + x^2 + x^4 + x^6 + \ldots$

$c_0 = 1$, $c_1 = 0$, $c_2 = 1$, $c_3 = 0$, $c_4 = 1$, $c_5 = 0$, $c_6 = 1$. $R = 1$.

3. $f(x) = \frac{x^2}{2-x} = x^2 \frac{1}{2-x} = \frac{x^2}{2} \cdot \frac{1}{1-\left(\frac{x}{2}\right)} = \frac{x^2}{2}\left(1 + \frac{x}{2} + \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^3 + \ldots\right) = \frac{x^2}{2} + \frac{x^3}{2^2} + \frac{x^4}{2^3} + \frac{x^5}{2^4} + \ldots$

$c_0 = 0$, $c_1 = 0$, $c_2 = \frac{1}{2}$, $c_3 = \frac{1}{4}$, $c_4 = \frac{1}{8}$, $c_5 = \frac{1}{16}$. $R = 2$.

converges iff $\left|\frac{x}{2}\right| < 1$, or $|x| < 2$.

$f(x) = 0 + 0 \cdot x + \frac{1}{2}x^2 + \frac{1}{4}x^3 + \frac{1}{8}x^4 + \frac{1}{16}x^5 + \ldots$