MAT 137 (Calculus II) Prof. Swift

The Taylor series for $f(x)$, centered at a, is $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$, where $c_n = \frac{f^{(n)}(a)}{n!}$.

1. Use the formula for c_n to find the Taylor series for the function $f(x) = x^4 + 3x^2 - 2x + 1$, centered at 0.

2. What is the radius of convergence of the Taylor series you found in question 1?

3. Use the formula for c_n to find the Taylor series for the same function, centered at 1.

1.

n	$f^{(n)}(x)$	$f^{(n)}(0)$	$c_n = \frac{f^{(n)}(0)}{n!}$
0	$f(x) = x^4+3x^2-2x+1$	1	$\frac{1}{0!} = \frac{1}{1} = 1$
1	$f'(x) = 4x^3+6x-2$	-2	$\frac{-2}{1!} = \frac{-2}{1} = -2$
2	$f''(x) = 12x^2+6$	$+6$	$\frac{+6}{2!} = \frac{+6}{2} = +3$
3	$f'''(x) = 24x$	0	$\frac{0}{3!} = 0$
4	$f''''(x) = 24$	24	$\frac{24}{4!} = \frac{24}{4\cdot3\cdot2\cdot1} = \frac{24}{24} = 1$
5	$f^{(5)}(x) = 0$	0	0
$n \geq 5$	$f^{(n)}(x) = 0$ for all x	0	0

So $f(x) = 1 - 2x + 3x^2 + 0x^3 + 1\cdot x^4 + 0\cdot x^5 + 0\cdot x^6 + \ldots$

$\boxed{f(x) = 1 - 2x + 3x^2 + x^4}$ The Taylor series is $f(x)$, and the taylor series terminates.

2. Since the Taylor series has only 4 nonzero terms, it converges for all x.

$\boxed{R = \infty}$

3.

n	$f^{(n)}(x)$	$f^{(n)}(1)$	$c_n = \frac{f^{(n)}(1)}{n!}$
0	$f(x) = x^4 + 3x^2 - 2x + 1$	$1+3-2+1 = 3$	$\frac{3}{0!} = 3$
1	$f'(x) = 4x^3 + 6x - 2$	$4+6-2 = 8$	$\frac{8}{1!} = 8$
2	$f''(x) = 12x^2 + 6$	$12+6 = 18$	$\frac{18}{2!} = 9$
3	$f'''(x) = 24x$	24	$\frac{24}{3!} = \frac{24}{6} = 4$
4	$f''''(x) = 24$	24	$\frac{24}{4!} = \frac{24}{24} = 1$
$n \geq 5$	$f^{(n)}(x) = 0$	0	0

So $\boxed{f(x) = 3 + 8(x-1) + 9(x-1)^2 + 4(x-1)^3 + (x-1)^4}$

(Again, $R = \infty$)

This is a useful alternative form of $f(x)$!