MAT 137 (Calculus II) Prof. Swift

Approximating $\sqrt{104}$ with a Taylor polynomial

Do not use a calculator for problems 1 and 2.

1. Find the degree 2 Taylor polynomial $T_2(x)$, centered at $a = 100$, for the function $f(x) = \sqrt{x} = x^{1/2}$. Leave fractions in your answer, and do not use decimals.

2. Use the fact that $f(x) \approx T_2(x)$ near $x = 100$ to approximate $\sqrt{104}$. Get an approximation as a sum involving fractions, then evaluate that approximation as an exact decimal. Your final answer will be a sentence, $\sqrt{104} \approx$ 10.198.

3. Use a calculator or the web to write $\sqrt{104}$, rounded to 7 significant figures.

1.

n	$f^{(n)}(x)$	$f^{(n)}(100)$	$c_n = \cancel{a_n} = \frac{f^{(n)}(100)}{n!}$
0	$f(x) = x^{\frac{1}{2}}$	$(100)^{\frac{1}{2}} = (10^2)^{\frac{1}{2}} = 10$	$\frac{10}{0!} = \frac{10}{1} = 10$
1	$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$	$\frac{1}{2}(10^2)^{-\frac{1}{2}} = \frac{1}{2}10^{-1} = \frac{1}{20}$	$\frac{\frac{1}{20}}{1!} = \frac{1}{20}$
2	$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}$	$-\frac{1}{4}(10^2)^{-\frac{3}{2}} = -\frac{1}{4}10^{-3} = \frac{-1}{4 \cdot 10^3}$	$\frac{\frac{-1}{4\cdot 10^3}}{2} = \frac{-1}{8 \cdot 10^3} = \frac{-1}{8000}$

So $T_2(x) = c_0 + c_1(x-100) + c_2(x-100)^2$

$$\boxed{T_2(x) = 10 + \frac{1}{20}(x-100) - \frac{1}{8\cdot 10^3}(x-100)^2}$$

2. $\sqrt{104} \approx T_2(104) = 10 + \frac{1}{20}\cdot 4 - \frac{1}{8\cdot 10^3}(4)^2$

$\sqrt{104} \approx 10 + \frac{4}{20} - \frac{16}{8\cdot 10^3} = \boxed{10 + \frac{1}{5} - \frac{2}{10^3}}$

Decimals: $\sqrt{104} \approx 10 + 0.2 - 0.002 = \boxed{10.198}$

3. Calculator says $\boxed{\sqrt{104} = 10.19804\ldots}$, rounded to 7 significant figures.