# MAT 137 (Calculus II) Prof. Swift

More Applications of Taylor Series

1. Starting with the first 3 nonzero terms in the Taylor series for $\sin(x)$, write out the first 2 nonzero terms in the Taylor series for $\frac{\sin(x)-x}{x^3}$. $= \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right) - x}{x^3} = \frac{-1}{3!} + \frac{x^2}{5!} - \dots$

2. Use the result of problem 1 to evaluate $\lim_{x\to 0} \frac{\sin(x)-x}{x^3}$, without using L'Hospital's.

3. Write out the first 3 nonzero terms of the Taylor series of $e^{-t^2}$.

4. Find the first 3 nonzero terms of the Taylor series of $F(x) = \int_0^x e^{-t^2}\,dt$.

5. Find the value of $F^{(5)}(0)$, using the answer to problem 4.

2. $$\lim_{x\to 0} \frac{\sin(x)-x}{x^3} = \lim_{x\to 0}\left(\frac{-1}{3!} + \frac{x^2}{5!} - \dots\right) = \frac{-1}{3!} = \boxed{\frac{-1}{6}}$$

3. $$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots, \text{ so } e^{-t^2} = 1 + (-t^2) + \frac{(-t^2)^2}{2!} + \dots$$

$$\boxed{e^{-t^2} = 1 - t^2 + \frac{t^4}{2} - \dots}$$

4. $$F(x) = \int_0^x e^{-t^2}\,dt = \int_0^x \left(1 - t^2 + \frac{t^4}{2} - \dots\right)dt$$

$$= t - \frac{t^3}{3} + \frac{t^5}{2\cdot 5} - \dots \Big|_0^x$$

$$\boxed{F(x) = x - \frac{x^3}{3} + \frac{x^5}{10} - \dots} = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots$$

5. $c_5 = \frac{F^{(5)}(0)}{5!}$ from the "formula of last resort"

and $c_5 = \frac{1}{10}$ from Question 4: so $\frac{F^{(5)}(0)}{5!} = \frac{1}{10}$

or $$F^{(5)}(0) = \frac{5!}{10} = \frac{\cancel{5}\cdot 4\cdot 3\cdot \cancel{2}\cdot 1}{\cancel{2}\cdot\cancel{5}} = 4\cdot 3 = \boxed{12.}$$