MAT 137 (Calculus II) Prof. Swift

Parametric Curves in the Plane, Part 2

1. Eliminate the parameter to get an equation in terms of x and y for the parabola with parametric description $x = 1+t^2$, $y = 3+t$. Write your answer in the form $x = Ay^2+By+C$.

2. Eliminate the parameter to get an equation in terms of x and y for the ellipse with parametric description $x = 3\cos(t)$, $y = 2\sin(t)$. Hint: $\sin^2(t) + \cos^2(t) = 1$ for all t.

3. The parametric equations $x = \sin(t) + \cos(t)$, $y = \sin(t)$, $0 \le t \le 2\pi$ trace out an ellipse. Find the points on this ellipse where the tangent line is horizontal. (Hint: The vertical velocity is zero when the curve has a horizontal tangent, so start by solving $\frac{dy}{dt} = 0$ for t.)

1. $y = 3+t$, so $t = y-3$

$x = 1+t^2$ becomes $x = 1+(y-3)^2 = 1+y^2-6y+9$

$$\boxed{x = y^2-6y+10}$$

2. $x = 3\cos(t)$ $\quad y = 2\sin(t)$

$\left(\frac{x}{3}\right)^2 = \cos^2(t)$ $\quad \left(\frac{y}{2}\right)^2 = \sin^2(t)$

$\therefore \left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = \cos^2(t)+\sin^2(t)$

$$\boxed{\frac{x^2}{9} + \frac{y^2}{4} = 1}$$

3. $x = \sin(t)+\cos(t)$, $0 \le t \le 2\pi$

$y = \sin(t)$

$\frac{dy}{dt} = \cos(t) \overset{\text{set}}{=} 0$: $t = \frac{\pi}{2}$ or $t = \frac{3\pi}{2}$

$t = \frac{\pi}{2}$: $x = \sin(\frac{\pi}{2})+\cos(\frac{\pi}{2}) = 1+0 = 1$

$y = \sin(\frac{\pi}{2}) = 1$ $\quad$ pt $= (1,1)$

$t = \frac{3\pi}{2}$: $x = \sin(\frac{3\pi}{2})+\cos(\frac{3\pi}{2}) = -1$

$y = \sin(\frac{3\pi}{2}) = -1$. $\quad$ pt $= (-1,-1)$.

$\boxed{\text{The two points where the tangent line is horizontal are } \pm(1,1).}$