

## MAT 137 (Calculus II) Prof. Swift

Quiz 8, Parametric Curves and Arc Length

Name: key

For this quiz, you *may* work with other people, and you *may* consult your notes. You may *not* use the internet. You may leave the class after you turn in your quiz.

For this page, a particle moves in the plane with position  $x = 3 \cos(t)$  and  $y = 2 \sin(t)$ .

1. Find the speed of the particle at time  $t$ . Denote the speed as  $v(t)$ , following the physics convention even though the speed is the *magnitude* of the velocity.

$$\vec{v} = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \langle -3 \sin(t), 2 \cos(t) \rangle$$

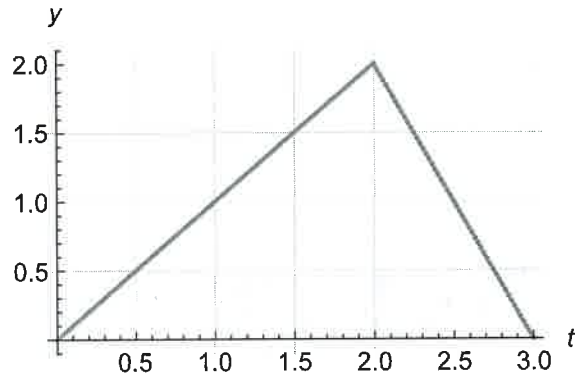
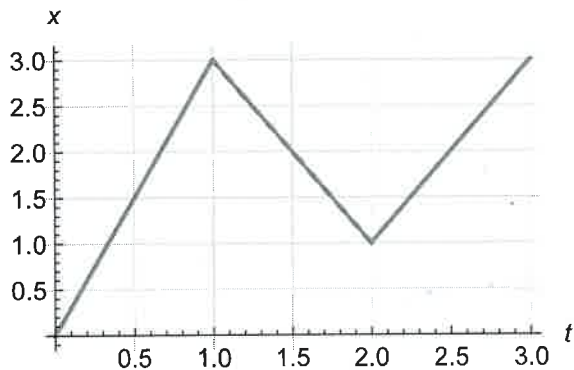
$$\text{So } v(t) = \sqrt{9 \sin^2(t) + 4 \cos^2(t)}$$

2. Write a definite integral for the distance traveled by the particle in the time interval  $0 \leq t \leq 2\pi$ . Note that this is also the perimeter of an ellipse.

$$\text{Distance traveled} = \text{Perimeter} = \int_0^{2\pi} \sqrt{9 \sin^2(t) + 4 \cos^2(t)} dt$$

(This is NOT an elementary integral.)

3. The top two figures show the graphs of  $x = f(t)$  and  $y = g(t)$ . On the bottom figure, sketch the parametric curve traced out for  $0 \leq t \leq 3$  in the  $x - y$  plane.



$t$	$x$	$y$	$(x, y)$
0	0	0	(0,0)
1	3	1	(3,1)
2	1	2	(1,2)
3	3	0	<del>(3,3)</del> (3,0)

