MAT 137 (Calculus II) Prof. Swift

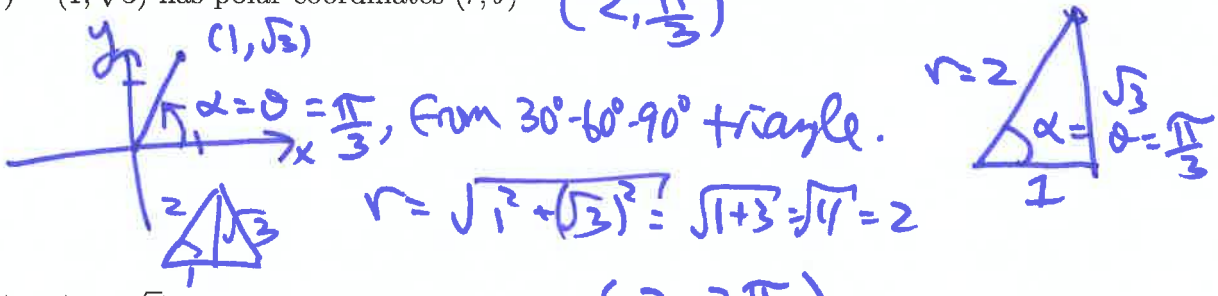
Polar Coordinates

It is easy to convert from polar to rectangular coordinates: $x = r\cos(\theta)$ and $y = r\sin(\theta)$.

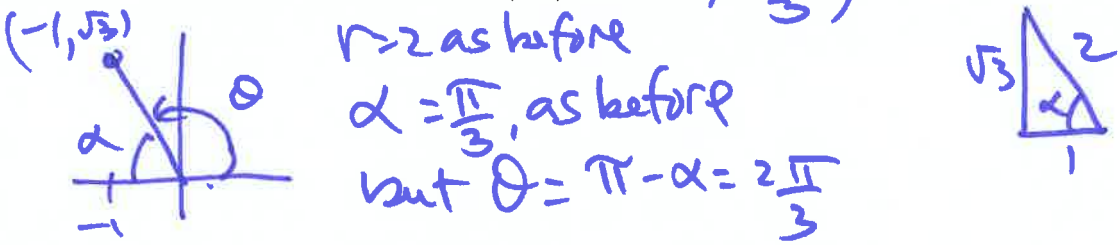
However, converting from rectangular to polar coordinates is tricky: $r^2 = x^2 + y^2$ and $\tan(\theta) = y/x$ (provided $x \neq 0$). If we are told we want $r \geq 0$, then choose $r = \sqrt{x^2 + y^2}$. However, note that $\theta = \arctan(y/x)$ *is only true in quadrant I.* To find θ, draw a picture of the point in the x-y plane! Consider what quadrant you are in. Let α = reference angle.

Convert these points to polar coordinates. If possible, give the unique answer with $r \geq 0$ and $0 \leq \theta < 2\pi$. Do not use inverse trig functions in the final answer, if possible.

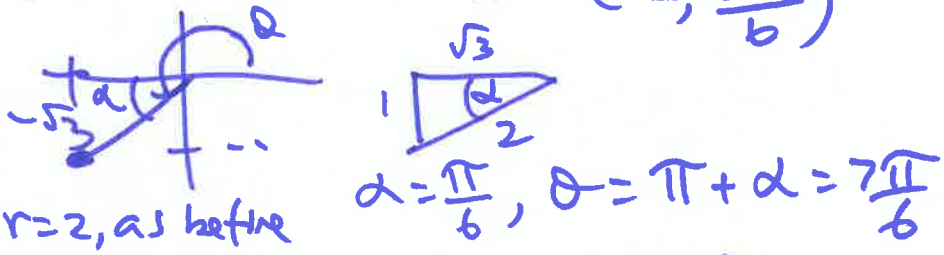
$(x, y) = (1, \sqrt{3})$ has polar coordinates $(r, \theta) =$ $(2, \frac{\pi}{3})$



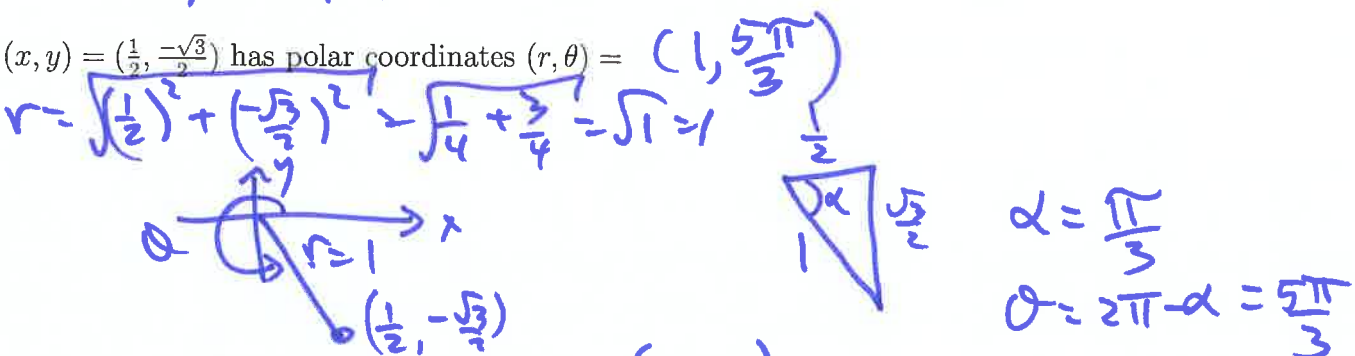
$(x, y) = (-1, \sqrt{3})$ has polar coordinates $(r, \theta) =$ $(2, \frac{2\pi}{3})$



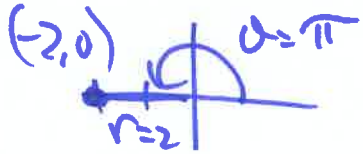
$(x, y) = (-\sqrt{3}, -1)$ has polar coordinates $(r, \theta) =$ $(2, \frac{7\pi}{6})$



$(x, y) = (\frac{1}{2}, \frac{-\sqrt{3}}{2})$ has polar coordinates $(r, \theta) =$ $(1, \frac{5\pi}{3})$

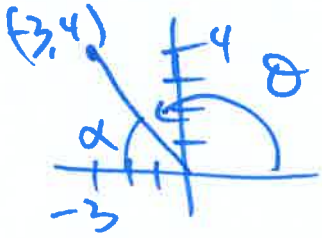


$(x, y) = (-2, 0)$ has polar coordinates $(r, \theta) =$ $(2, \pi)$



$(x, y) = (-3, 4)$ has polar coordinates $(r, \theta) =$ $(5, \pi - \arctan(\frac{4}{3}))$

$r = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$ (3-4-5 triangle)



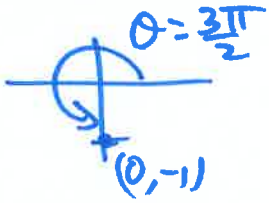
$\alpha = \arctan(\frac{4}{3})$

$\theta = \pi - \alpha$

$= \pi - \arctan(\frac{4}{3})$

$(x, y) = (0, -1)$ has polar coordinates $(r, \theta) =$ $(1, \frac{3\pi}{2})$

$r = \sqrt{0^2 + (-1)^2} = \sqrt{1} = 1$

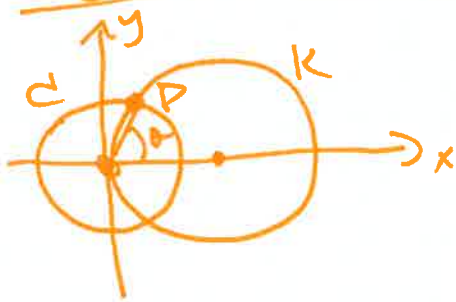


$(x, y) = (-2, -2)$ has polar coordinates $(r, \theta) =$ $(2\sqrt{2}, \frac{\pi}{4})$

$r = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$\theta = \frac{5\pi}{4}$, by inspection: or. $\alpha = \frac{\pi}{4}$, $\theta = \pi + \alpha$.

Figure for Problem



$(x, y) = (3, 0)$ has polar coordinates $(r, \theta) =$ $(3, 0)$

$r = 3, \theta = 0$

$(x, y) = (0, 0)$ has polar coordinates $(r, \theta) =$ $(0,0)$, or $(0,1)$, or $(0, \frac{\pi}{4})$, etc.

$r = 0$, $\theta =$ anything $= (0,$ undefined$)$

or $\theta =$ undefined. $= (0,$ anything$)$

$\vdots$

(This has many answers. Now you see why I said, "*If possible* give the unique answer ...".)