MAT 216 (Introduction to Matrix Algebra), Prof. Jim Swift
Workheet 3: Gaussian Elimination and Gauss-Jordan Elimination

Do Gaussian elimination to put the following augmented matrix into row-echelon form. Then, continue and put the augmented matrix into reduced row-echelon form. (This is called Gauss-Jordan elimination.) Then, write down the solution of the system of linear equations with the original augmented matrix. Assume the unknowns are (x, y, z).

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$$\rightsquigarrow -\tfrac{1}{3} R_3 \to R_3 \left[\begin{array}{ccc|c} 1 & 2 & 0 & -4 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Row Echelon Form!

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Row Echelon Form!

$$\rightsquigarrow R_1 - 2R_2 \to R_1 \left[\begin{array}{ccc|c} 1 & 0 & 4 & 6 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

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Row Echelon Form!

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Row-Echelon Form! (REF)

$$\rightsquigarrow R_1 - 2R_2 \to R_1 \left[\begin{array}{ccc|c} 1 & 0 & 4 & 6 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

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Reduced Row-Echelon Form (RREF)

The solution is $(x, y, z) = (2, -3, 1)$