

MAT 216 (Introduction to Matrix Algebra), Prof. Jim Swift
 Worksheet 4 = Quiz 2: Systems of Linear Equations

Name: key

1. A system of 3 linear equations in 3 unknowns has an augmented matrix that can be reduced to $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{array} \right]$. The solution of the system is $(x, y, z) = (3-2z, 5-4z, z)$

\uparrow
 z is free

$$\begin{aligned} x + 2z &= 3 \quad \text{or} \quad x = 3 - 2z \\ y + 4z &= 5 \quad \quad \quad y = 5 - 4z \end{aligned}$$

2. Consider the system of linear equations

$$\begin{aligned} x + y &= 5 \\ x - 2y &= -4 \end{aligned}$$

(a) Write down the augmented matrix of the system.

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 1 & -2 & -4 \end{array} \right]$$

(b) Reduce the augmented matrix to reduced row-echelon form.

$$\begin{array}{l} \rightsquigarrow \\ R_2 - R_1 \rightarrow R_2 \end{array} \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & -3 & -9 \end{array} \right]$$

$$\begin{array}{l} \rightsquigarrow \\ -\frac{1}{3}R_2 \rightarrow R_3 \end{array} \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 1 & 3 \end{array} \right]$$

$$\rightsquigarrow R_1 - R_2 \rightarrow R_1 \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \end{array} \right]$$

(c) Write down the solution of the system. $(x, y) = (2, 3)$.