

MAT 216 (Introduction to Matrix Algebra), Prof. Jim Swift
Worksheet 5 = Quiz 3: Inverse Matrices

Name: key

1. Use the row-reduction method to find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

row reduce $[A | I]$ to get $[I | A^{-1}]$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 - 2R_2 \rightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & -5 & 1 & -2 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 + 5R_3 \rightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 5 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{R_2 - 4R_3 \rightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 5 \\ 0 & 1 & 0 & 0 & 1 & -4 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

This is very error prone, so you should check!

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0+0 & -2+2+0 & 5-8+3 \\ 0+0+0 & 0+1+0 & 0-4+4 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ yes!}$$

2. Find the matrix A , assuming that $A^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$. You may use the known inverse of a 2x2 matrix, but you do not have to.

$$A = (A^{-1})^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}^{-1} \quad \text{use } \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad \leftarrow \text{many used the wrong formula}$$

$$A = \frac{1}{5-6} \begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix} = - \begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$$

$$\text{Check: } \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} -5+6 & 2-2 \\ 15+5 & 6-5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ yes!}$$