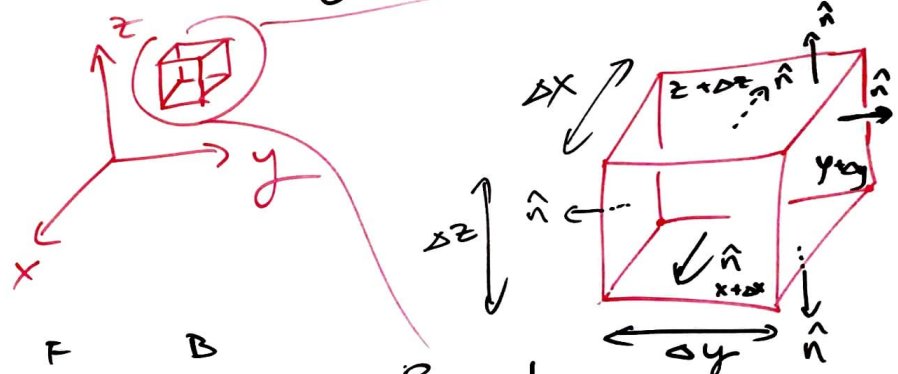
Q. What is the Total Flux of $\vec{F}(x,y,z)$ out of a tiny box $[x, x+\Delta x] \times [y, y+\Delta y] \times [z, z+\Delta z]$?



$$\iint_{\text{Box}} \vec{F}\cdot d\vec{s} \approx F_1(x+\Delta x, y, z)\cdot \Delta y \Delta z \quad \text{Front}$$
$$- F_1(x, y, z)\,\Delta y \Delta z \quad \text{back}$$
$$+ F_2(x, y+\Delta y, z)\,\Delta x \Delta z \quad \text{right}$$
$$- F_2(x, y, z)\,\Delta x \Delta z \quad \text{left}$$
$$+ F_3(x, y, z+\Delta z)\,\Delta x \Delta y \quad \text{TOP}$$
$$- F_3(x, y, z)\,\Delta x \Delta y \quad \text{bottom}$$

So

$$\iint_{\text{Tiny Box}} \vec{F}\,d\vec{S} \approx \left[\frac{F_1(x+\Delta x,y,z)-F_1(x,y,z)}{\Delta x} + \frac{F_2(x,y+\Delta y,z)-F_2(x,y,z)}{\Delta y} + \frac{F_3(x,y,z+\Delta z)-F_3(x,y,z)}{\Delta z}\right]\Delta x\cdot\Delta y\cdot\Delta z$$

$$\approx \left(\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}\right)\Delta x\Delta y\Delta z = \left(\vec{\nabla}\cdot\vec{F}\right)\Delta x\Delta y\Delta z \leftarrow \text{Answer}$$