

MAT 136, Prof. Swift

Differentiation Shortcuts

Rules:

$$\frac{d}{dx} [c \cdot f(x)] = c \cdot f'(x) \quad \text{The Constant Multiple Rule (} c \text{ is any constant).} \quad (cf)' = cf'$$

$$\frac{d}{dx} [f(x) \pm g(x)] = f'(x) \pm g'(x) \quad \text{The Sum and Difference Rules.} \quad (f \pm g)' = f' \pm g'$$

$$\frac{d}{dx} [f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x) \quad \text{The Product Rule.} \quad (f \cdot s)' = f' \cdot s + f \cdot s'$$

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2} \quad \text{The Quotient Rule.} \quad \left(\frac{t}{b} \right)' = \frac{t' \cdot b - t \cdot b'}{b^2}$$

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x) \quad \text{The all-important Chain Rule.} \quad (f \circ g)' = (f' \circ g) \cdot g'$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad \text{The all-important Chain Rule in Leibnitz notation.}$$

Facts:

$$\frac{d}{dx} [m \cdot x + b] = m \quad \text{for any constants } m \text{ and } b. \text{ This implies } \frac{d}{dx} c = 0, \text{ so } \frac{d}{dx} [f(x) + c] = f'(x).$$

$$\frac{d}{dx} x^c = c \cdot x^{c-1} \quad \text{for any constant } c \neq 0 \text{ (positive, negative, integer, fraction, or irrational).}$$

$$\frac{d}{dx} e^x = e^x \quad \text{and more generally } \frac{d}{dx} a^x = \ln(a) \cdot a^x \text{ for any positive constant } a.$$

$$\frac{d}{dx} \sin(x) = \cos(x) \quad \frac{d}{dx} \cos(x) = -\sin(x) \quad \frac{d}{dx} \tan(x) = \frac{1}{\cos^2(x)} = \sec^2(x)$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x} \quad \text{with domain } x > 0 \quad \frac{d}{dx} \ln|x| = \frac{1}{x} \quad \text{with domain } x \neq 0$$

$$\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$$

The Trigonometric Identities you need to know:

$$\sin^2(x) + \cos^2(x) = 1 \quad \tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\csc(x) = \frac{1}{\sin(x)} \quad \sec(x) = \frac{1}{\cos(x)} \quad \cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$$

The Logarithm Identities you need to know:

$$\ln(a) = b \Leftrightarrow e^b = a, \text{ so } \ln(1) = 0 \text{ (since } e^0 = 1), \text{ and } \ln(e) = 1 \text{ (since } e^1 = e), \text{ etc.}$$

$$e^{\ln(x)} = x \text{ for all } x > 0 \quad \ln(e^x) = x \text{ for all } x \quad \log_a(x) = \ln(x)/\ln(a) \quad a^x = e^{\ln(a) \cdot x}$$

$$\ln(ab) = \ln(a) + \ln(b) \quad \ln(a/b) = \ln(a) - \ln(b) \quad \ln(a^b) = b \ln(a)$$