$\nabla f(r,h) = \lambda \nabla g(r,h)$ becomes

$$\langle 2\pi h + 4\pi r, 2\pi r \rangle = \lambda \langle 2\pi r h, \pi r^2 \rangle$$

$$= \langle 2\pi r h \lambda, \pi r^2 \lambda \rangle$$

$$\boxed{\begin{cases} 2\pi h + 4\pi r = 2\pi r h \lambda \\ 2\pi r = \pi r^2 \lambda \\ \pi r^2 h = 1500 \end{cases}}$$

of corse $r \neq 0$

$2\pi r = \pi r^2 \lambda$

$2 = r\lambda$

so $\lambda = \frac{2}{r}$

$2\pi h + 4\pi r = 2\pi r h \left(\frac{2}{r}\right)$

$= 4\pi h$

$4\pi r = 4\pi h - 2\pi h = 2\pi h$

$2r = h$ for any fixed volume!

Plug $h = 2r$ into $\pi r^2 h = 1500$

$\pi r^2 (2r) = 1500$

$2\pi r^3 = 1500$

$r^3 = \frac{750}{\pi}$

$$\boxed{r = \left(\frac{750}{\pi}\right)^{\frac{1}{3}}, \quad h = 2\left(\frac{750}{\pi}\right)^{\frac{1}{3}}}$$ ← Final Answer!

Let $f(r,h)$ be the Surface Area of the Cylinder (in cm^2)

Let $g(r,h)$ be the volume of the cylinder (in cm^3)

minimize $f(r,h)$ subject to $g(r,h)=1500$

$f(r,h) = 2\pi rh + \pi r^2 + \pi r^2 = 2\pi rh + 2\pi r^2$; $g(r,h) = \underbrace{\pi r^2}_{\text{area of base}} h$

use: $\nabla f = \lambda \nabla g$, $\nabla f(r,h) = \langle 2\pi h + 4\pi r, 2\pi r \rangle$

$g = 1500$, $\nabla g(r,h) = \langle 2\pi rh, \pi r^2 \rangle$