

MAT 137 (Calculus II) Prof. Swift

Polar Coordinates

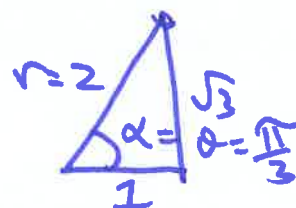
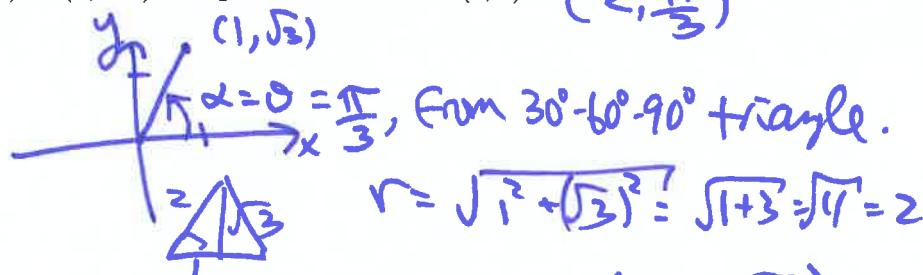
It is easy to convert from polar to rectangular coordinates: $x = r \cos(\theta)$ and $y = r \sin(\theta)$.

However, converting from rectangular to polar coordinates is tricky: $r^2 = x^2 + y^2$ and $\tan(\theta) = y/x$ (provided $x \neq 0$). If we are told we want $r \geq 0$, then choose $r = \sqrt{x^2 + y^2}$.

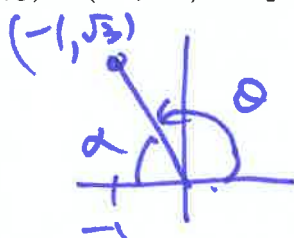
However, note that $\theta = \arctan(y/x)$ is only true in quadrant I. To find θ , draw a picture of the point in the x - y plane! Consider what quadrant you are in. *Let α = reference angle.*

Convert these points to polar coordinates. If possible, give the unique answer with $r \geq 0$ and $0 \leq \theta < 2\pi$. Do not use inverse trig functions in the final answer, if possible.

$(x, y) = (1, \sqrt{3})$ has polar coordinates $(r, \theta) = (2, \frac{\pi}{3})$



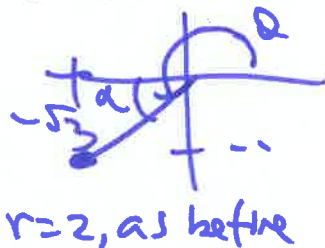
$(x, y) = (-1, \sqrt{3})$ has polar coordinates $(r, \theta) = (2, \frac{2\pi}{3})$



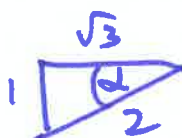
$r = 2$ as before
 $\alpha = \frac{\pi}{3}$, as before
 but $\theta = \pi - \alpha = \frac{2\pi}{3}$



$(x, y) = (-\sqrt{3}, -1)$ has polar coordinates $(r, \theta) = (2, \frac{7\pi}{6})$



$r = 2$, as before
 $\alpha = \frac{\pi}{6}$, $\theta = \pi + \alpha = \frac{7\pi}{6}$



$(x, y) = (\frac{1}{2}, -\frac{\sqrt{3}}{2})$ has polar coordinates $(r, \theta) = (1, \frac{5\pi}{3})$

$r = \sqrt{(\frac{1}{2})^2 + (-\frac{\sqrt{3}}{2})^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$

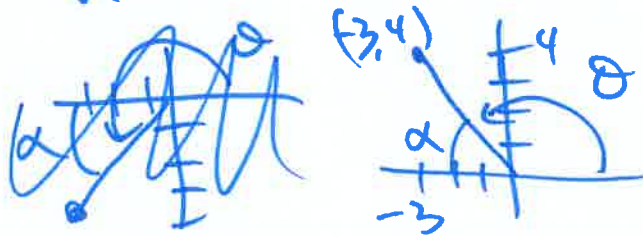


$\alpha = \frac{\pi}{3}$
 $\theta = 2\pi - \alpha = \frac{5\pi}{3}$

$(x, y) = (-2, 0)$ has polar coordinates $(r, \theta) = (2, \pi)$

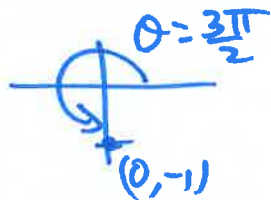


$(x, y) = (-3, 4)$ has polar coordinates $(r, \theta) = \boxed{(5, \pi - \arctan(\frac{4}{3}))}$
 $r = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ (3-4-5 triangle)



$\alpha = \arctan(\frac{4}{3})$
 $\theta = \pi - \alpha$
 $= \pi - \arctan(\frac{4}{3})$

$(x, y) = (0, -1)$ has polar coordinates $(r, \theta) = (1, \frac{3\pi}{2})$
 $r = \sqrt{0^2 + (-1)^2} = \sqrt{1} = 1$



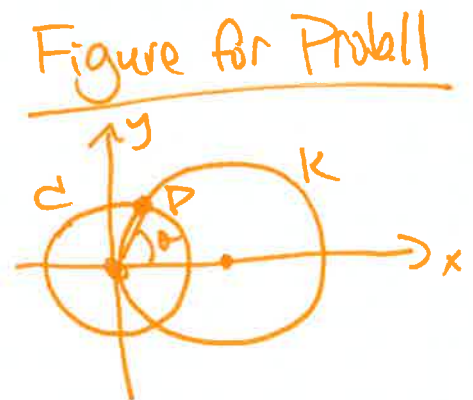
$(x, y) = (-2, -2)$ has polar coordinates $(r, \theta) = (2\sqrt{2}, \frac{5\pi}{4})$
 $r = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$



$\theta = \frac{5\pi}{4}$, by inspection: or. $\alpha = \frac{\pi}{4}$, $\theta = \pi + \alpha$.



$(x, y) = (3, 0)$ has polar coordinates $(r, \theta) = (3, 0)$



$(x, y) = (0, 0)$ has polar coordinates $(r, \theta) = (0, 0)$, or $(0, 1)$, or $(0, \frac{\pi}{4})$, etc.

$r = 0$, $\theta = \text{anything}$
 or $\theta = \text{undefined}$.
 $= (0, \text{undefined})$
 $= (0, \text{anything})$
 $\vdots$

(This has many answers. Now you see why I said, "If possible give the unique answer ...".)