How to solve an Exact ODE

Step 1. Put the ODE in "differential form"

$M(x,y)\,dx + N(x,y)\,dy = 0$, or $M(x,y) + N(x,y)\frac{dy}{dx} = 0$.

Step 2. Is this $\psi_x(x,y)\,dx + \psi_y(x,y)\,dy = 0$ in disguize?
Is ODE exact? (same question). Is this $d\psi = 0$ in disguize?
Is $M_y(x,y) = N_x(x,y)$ for all x, y? (same question)

↘ NO → Stop

↙ Yes

Step 3. ① $\psi_x(x,y) = M(x,y)$ $\Rightarrow$ $\psi(x,y) = \int M(x,y)\,dx + h(y)$ (*)

② $\psi_y(x,y) = N(x,y)$ ↘ ↓

$M(x,y) + h'(y) = N(x,y)$ "constant of integration"

Step 4. You should get $h'(y)$ = function of y only, after canceling x terms.
Solve for $h(y)$ by integrating, then plug this $h(y)$ into (*)

Step 5. "General" solution is $\psi(x,y) = C$. Step 6. If IC is given, plug in x_0, y_0 to find C.