

MAT 239 (Differential Equations) Solutions to handout on Classification of Differential Equations

Consider the differential equation: $\frac{dQ}{dt} = -kQ$

- Q What is the dependent variable?
 - t What is/are the independent variable(s)?
 - ODE Is this an ODE or a PDE?
 - Yes Is the DE linear?
 - 1 What is the order of the DE?
 - No Is $Q = 3e^{kt}$ a solution of the DE? If not, can you guess a solution?
- $Q = 3e^{-kt}$ and $Q = e^{-kt}$ and $Q = Q_0e^{-kt}$ for any Q_0 are all solutions.

Consider the differential equation: $(1 - x^2)y'' - 2xy' + 2y = 0$

- y What is the dependent variable?
- x What is/are the independent variable(s)?
- ODE Is this an ODE or a PDE?
- Yes Is the DE linear? *Note: DE is linear in the dependent variable, y .*
- 2 What is the order of the DE?
- Yes Is $y = x$ a solution of the DE? If not, can you guess a solution?

Consider the differential equation: $u_t + uu_x = 0$

- u What is the dependent variable?
- t and x What is/are the independent variable(s)?
- PDE Is this an ODE or a PDE? *Note: there is more than one independent variable.*
- No Is the DE linear? *Note: the uu_x term counts like u^2 , making it nonlinear*
- 1 What is the order of the DE?
- Yes Is $u = 0$ a solution of the DE? If not, can you guess a solution?

Consider the differential equation: $\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin(\theta)$

- θ What is the dependent variable?
- t What is/are the independent variable(s)?
- ODE Is this an ODE or a PDE?
- No Is the DE linear? *Note: the $\sin(\theta)$ term is nonlinear in θ .*
- 2 What is the order of the DE?
- No Is $\theta = \frac{1}{2}gt^2$ a solution of the DE? If not, can you guess a solution?

The only solutions I can write down in closed form are $\theta = 0$ and $\theta = \pi$ (or any integer multiple of π).