

# How to solve a 1<sup>st</sup> order Linear ODE

Step 1: Put ODE in standard form:  $y' + p(x)y = g(x)$

Step 2: Compute "magic" integrating factor and simplify.

$$\mu(x) = \exp\left(\int p(x) dx\right)$$

Step 3: multiply both sides of standard form by  $\mu(x)$ .

$$\mu(x)y' + \mu(x)p(x)y = \mu(x)g(x)$$

Step 4: Rewrite the L.H.S. as  $\frac{d}{dx}(\mu(x)y)$ . Check using product rule

$$\frac{d}{dx}(\mu(x)y) = \mu(x)g(x)$$

Step 5: Integrate to get  $\mu(x)y$ , and solve for  $y$ .

Step 6: If I.C. is given, solve for  $C$  and substitute  $C$  into general solution

General Solution

$$y = \frac{\int \mu(x)g(x) dx + C}{\mu(x)}$$