MAT 239 (Differential Equations), Prof. Swift
Worksheet 13, Exam Review

1. Put a "Y" or "N" in the blank, indicating if the ODE has the indicated property. Use the differential form to test exactness. Also, decide how you would find the general solution most easily, or decide that you should punt on this one.

N Separable

Y Linear

N Exact

$$\frac{dy}{dx} = 3x^2y + x \text{ or } (3x^2y + x)dx - dy = 0.$$

Solve as a linear ODE Compute $\mu(x)$, etc.

$$\frac{\partial}{\partial y}(3x^2y+x) \stackrel{?}{=} \frac{\partial}{\partial x}(-1)$$

$$3x^2 \neq 0$$ NOT exact

N Separable

N Linear

N Exact

$$\frac{dy}{dx} = x^2 - y^2 \text{ or } (y^2 - x^2)dx + dy = 0.$$

Punt! we have no way to solve this, with pencil & paper.

$$\frac{\partial}{\partial y}(y^2-x^2) \stackrel{?}{=} \frac{\partial}{\partial x}(1)$$

$$2y \neq 0$$ NOT exact

N Separable

N Linear

Y Exact

$$\frac{dy}{dx} = -\frac{x^2 + 2xy + y^2}{x^2 + 2xy} \text{ or } (x^2 + 2xy + y^2)dx + (x^2 + 2xy)dy = 0.$$

$$\frac{\partial}{\partial y}(x^2+2xy+y^2) \stackrel{?}{=} \frac{\partial}{\partial x}(x^2+2xy)$$

$$2x+2y = 2x+2y$$ Exact

2. Solve the IVP $\frac{dy}{dt} = y^2, \quad y(0) = y_0$, where y_0 is a positive constants. Find the interval of existence of the solution, and sketch the solution.

ODE is separable.

$$\frac{dy}{y^2} = dt$$

$$\int y^{-2}\,dy = \int dt$$

$$\frac{y^{-1}}{-1} = t + c \quad \text{(Add c NOW.)}$$

$$\frac{-1}{y} = t + c. \quad \cancel{\text{Now plug in } y(0)=y_0}.$$

$$y = \frac{-1}{t+c} \quad \text{is "general" solution}$$

Plug in $y(0) = y_0$: $y_0 = \frac{-1}{0+c}$, so $c = \frac{-1}{y_0}$

$$\boxed{y = \frac{-1}{t - \frac{1}{y_0}} = \frac{1}{\frac{1}{y_0} - t} = \frac{y_0}{1 - y_0 t}}$$

Any of these are OK.

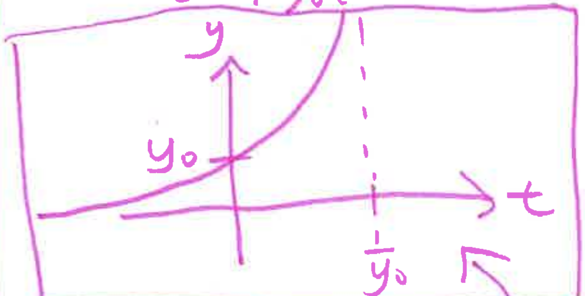
The interval of existence is the largest interval containing 0 on which the solution $y(t)$ is defined.

Bad value of t: $t - \frac{1}{y_0} = 0$

or $t = \frac{1}{y_0} > 0$.

$$\boxed{\text{Interval of existence is } t < \frac{1}{y_0}, \text{ or } \left(-\infty, \frac{1}{y_0}\right)}$$

Sketch $y = \frac{y_0}{1 - y_0 t}$ for $t < \frac{1}{y_0}$



Nothing here!