

MAT 239 (Differential Equations), Prof. Swift
The Method of Undetermined Coefficients (5 class points)

Work on this with a group of 3 or 4 people. Turn in *one* sheet for each group. Each person should copy (and sign) the group's solution to at least one problem.

1. Find the general solution to $y'' - y' - 2y = 0$, where x is the independent variable.

$$r^2 - r - 2 = 0$$

$$(r+1)(r-2) = 0$$

$$r_1 = -1, r_2 = 2$$

$$y = c_1 e^{-x} + c_2 e^{2x}$$

2. Find the general solution to $y'' - y' - 2y = e^{3x}$.

$y = y_h + y_p$

$y_h = \text{answer to 1.}$

$y_p = A e^{3x}$

$y_p' = 3A e^{3x}$

$y_p'' = 9A e^{3x}$

$y_p = \frac{1}{4} e^{3x}, \text{ so}$

$$y = c_1 e^{-x} + c_2 e^{2x} + \frac{1}{4} e^{3x}$$

$$9A e^{3x} - (3A e^{3x}) - 2(A e^{3x}) = e^{3x}$$

$$(9A - 3A - 2A) e^{3x} = e^{3x}$$

$$4A = 1, A = \frac{1}{4}$$

3. Find the general solution to ODE $y'' + y' = x$.

$$r^2 + r = 0$$

$$r(r+1) = 0$$

$$r = 0, -1$$

First guess

$$e^0 = 1, e^{-1x} = e^{-x}, y_h = C_1 + C_2 e^{-x}$$

$y_p(x) = Ax + B$, But $y = B$ is included in the y_h solution

$y_p(x) = x(Ax + B) = Ax^2 + Bx$ is the form.

$$y_p' = 2Ax + B$$

$$y_p'' = 2A$$

$$2A + (2Ax + B) = x$$

$$(2A + B) + 2Ax = x$$

$$\text{so } \begin{cases} 2A + B = 0 \\ 2A = 1 \end{cases} \quad A = \frac{1}{2}$$
$$2\left(\frac{1}{2}\right) + B = 0; B = -1$$

$$y_p = \frac{1}{2}x^2 - x$$

the general solution is

$$y = y_h + y_p = C_1 + C_2 e^{-x} + \frac{1}{2}x^2 - x$$