MAT 239 (Differential Equations), Prof. Swift
Worksheet 6 on Linear 1st Order ODEs

In this worksheet you will find the general solution to $xy' + 3y = 5x^2$.

0. Is $y(x) = 0$ a solution? yes/no Is $y(x) = k$ a solution for any constant k? yes/no

$0 + 0 \neq 5x^2$ $\qquad$ $0 + 3k \neq 5x^2$

1. Put the ODE $xy' + 3y = 5x^2$ into standard form and identify $p(x)$ and $g(x)$. What is the x value where p and/or q are not continuous?

$y' + \frac{3}{x} y = 5x$, so $p(x) = \frac{3}{x}$ and $g(x) = 5x$

p is NOT continuous (not even defined) at $\underline{x=0}$.

2. Compute *and simplify* the "magic" integrating factor $\mu(x)$

$$\mu(x) = \exp\left(\int \frac{3}{x}\, dx\right) = \exp(3 \ln(x) + C)$$

$$= e^{\ln(x^3) + C} = e^{\ln(x^3)} \cdot e^C$$

$= x^3 \cdot e^C$. We didn't need C! $\boxed{\mu(x) = x^3}$

(choose $C = 0$)

2b. Compute $\frac{d}{dx}[\mu(x)y]$ after substituting the formula you found for $\mu(x)$, but leaving y as an unknown function of x.

$$\frac{d}{dx}[x^3y] = 3x^2y + x^3y'$$

3. Multiply both sides of the *standard form* of the ODE by $\mu(x)$.

$$x^3y' + 3x^2y = 5x^4$$

4. Note that the Left Hand Side (LHS) of that ODE is what you computed in part 2b. (This is the magic of $\mu(x)$.) Rewrite the ODE as $\frac{d}{dx}[\mu(x)y] = \mu(x)g(x)$.

$$\frac{d}{dx}[x^3y] = 5x^4$$

5 Integrate to find $\mu(x)y$ (don't forget the "+C") and then solve for y. This is the general solution!

$$x^3y = \int 5x^4\,dx = x^5 + C$$

$$\boxed{y = \frac{x^5+C}{x^3} = x^2 + \frac{C}{x^3}}$$

Either one is OK.