# MAT 239 (Differential Equations), Prof. Swift
## Worksheet 7 on Linear 1st Order ODEs

A common IVP in applications has the form $\frac{dy}{dt} = -2y + 6, \quad y(0) = 0$. ($t$ is time.)

0. Is $y(t) = 0$ a solution to the ODE? yes/no Is $y(t) = k$ a solution to the ODE for some constant $k$? yes/no. If so, write down the constant solution.

$0 \neq 2 \cdot 0 + 6$ $\quad$ $0 = 2k - 6$ $\quad$ $\boxed{y(t) = 3}$ is a constant solution

$k = 3$, so

1. Put the ODE into standard form and identify $p(t)$ and $g(t)$. A theorem says that the particular solution is defined for all $t$, since $p$ and $q$ are continuous for all $t$.

$y' + 2y = 6 \quad p(t) = 2, \; g(t) = 6.$

2. Follow the recipe for 1st order linear ODEs to find the general solution.

$\mu(t) = \exp\left(\int 2\,dt\right) = e^{2t + \cancel{c}} = e^{2t} \quad (\mu(t) = e^{2t})$

$e^{2t} y' + 2e^{2t} y = 6e^{2t}$

$\frac{d}{dt}(e^{2t} y) = 6e^{2t}$

$e^{2t} y = \int 6e^{2t}\,dt = 3e^{2t} + c$

$$\boxed{y = \frac{3e^{2t} + c}{e^{2t}} = 3 + ce^{-2t}}$$

OK $\quad$ much better

3. Find the particular solution to the IVP, and sketch the solution for $t \geq 0$ without a calculator. Draw a dotted line at the horizontal asymptote. On the axes, indicate $y = 0$, $y = 3$, $t = 0$, and the approximate position of $t = \frac{1}{2}$. (Hint: $e = 2.718\ldots \approx 3$.)

$y(0) = 0$, so plug in $t = 0$, $y = 0$ into

$y = 3 + Ce^{-2t}$

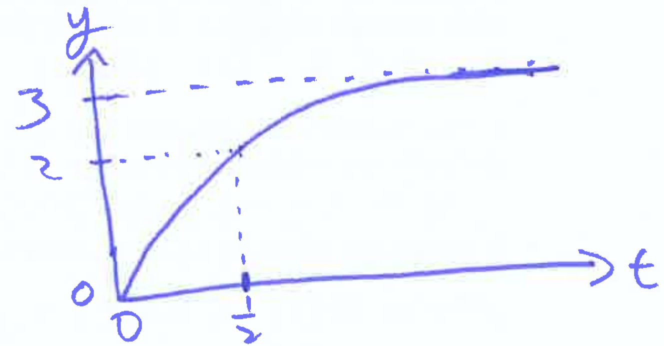
$0 = 3 + C\cancel{e^{0}}^{\,1}$

$C = -3$

so

$$\boxed{y = 3 - 3e^{-2t}}$$

Particular solution




Note that

$y(\tfrac{1}{2}) = 3 - 3e^{-2(\frac{1}{2})}$

$= 3 - 3e^{-1}$

$= 3 - \frac{3}{e}$ ← $e \approx 3$

$y(\tfrac{1}{2}) \approx 3 - 1 = 2$

$y(\frac{1}{2}) \approx 2$ tells us where to put the tick mark for $t = \frac{1}{2}$.