7. In this problem we will model pollution of a lake. Assume that the volume of the lake is 10^6 cubic meters. Initially the water in the lake is pure, but at time $t = 0$ a trans fat factory starts polluting the lake by pumping in effluent with a concentration of 2 kilograms per cubic meter of "substance F". The effluent is pumped in at a rate of 100 cubic meters per day. There is also a source of pure water entering the lake at a rate of 900 cubic meters per day. The pollutant in the lake is well mixed, and the mixed water drains out of the lake at the rate of 1000 cubic meters per day. (Thus, the volume of the lake stays at a constant 10^6 cubic meters.)

(a) Write down the IVP for $y(t)$, the kilograms of substance F in the lake after the factory has been open t days.
(b) Solve the IVP. (You may solve by inspection.)
(c) How many kilograms of substance F are in the lake in the limit $t \to \infty$?

$$\frac{dy}{dt} = \text{rate in} - \text{rate out} \quad \left(\frac{kg}{day}\right)$$

$$\text{rate in} = \frac{2\,kg}{m^3} \cdot \frac{100\,m^3}{day} = 200\,\frac{kg}{day}$$

$$\text{rate out} = \frac{y}{10^6}\frac{kg}{m^3} \cdot 10^3\,\frac{m^3}{day} = \frac{y}{10^3}\,\frac{kg}{day}$$

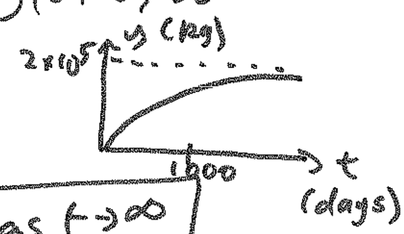
$$\boxed{y(0) = 0}$$

$$\boxed{\frac{dy}{dt} = 200 - \frac{y}{10^3}} = \frac{-y}{10^3} + \frac{10^3}{10^3} \cdot 200$$

$$= \frac{-1}{10^3}\left(y - 2\times10^5\right)$$

$$y = 2\times10^5 + C e^{-\frac{t}{10^3}}$$

$y(0) = 0$, so



$$\boxed{y = 2\times10^5\left(1 - e^{-\frac{t}{10^3}}\right)}$$

$\boxed{2\times10^5 \text{ kg of Substance F as } t \to \infty}$