MAT 239 (Differential Equations), Prof. Swift Worksheet 10 on Autonomous 1st Order ODEs

The population, P(t), of green-eyed frogs on an island (measured in thousands of frogs) at time t (measured in months) obeys the differential equation

$$\frac{dP}{dt} = f(P),$$

where the graph of f is shown below.



(a) Write down the equilibrium solutions (also called constant solutions) of this ODE.

(b) On the back of this page, sketch the graph of several solutions, P(t), of this ODE. Be sure to include the equilibrium solutions. Put a few numbered tick marks on the t axis to indicate the approximate time scale. (Hint: The maximum value of $\frac{dP}{dt}$ is 0.2 thousand frogs per month.)

MAT 239 (Differential Equations), Prof. Swift Worksheet 10 on Autonomous 1st Order ODEs

The population, P(t), of green-eyed frogs on an island (measured in thousands of frogs) at time t (measured in months) obeys the differential equation

$$\frac{dP}{dt} = f(P),$$

where the graph of f is shown below.



(a) Write down the equilibrium solutions (also called constant solutions) of this ODE.

(b) On the back of this page, sketch the graph of several solutions, P(t), of this ODE. Be sure to include the equilibrium solutions. Put a few numbered tick marks on the t axis to indicate the approximate time scale. (Hint: The maximum value of $\frac{dP}{dt}$ is 0.2 thousand frogs per month.)



(c) Based on the graph of the solutions, which of the equilibrium solutions are stable, and which are unstable. (Hint: Nearby solutions approach a stable equilibrium and diverge away from an unstable equilibrium.)

(d) Fill in the blanks. If the initial population of green-eyed frogs, P(0), is below ______ thousand, then the frogs will go extinct. On the other hand, if the initial population is above ______ thousand frogs, then the frog population will eventually reach approximately ______ thousand.



(c) Based on the graph of the solutions, which of the equilibrium solutions are stable, and which are unstable. (Hint: Nearby solutions approach a stable equilibrium and diverge away from an unstable equilibrium.)

(d) Fill in the blanks. If the initial population of green-eyed frogs, P(0), is below ______ thousand, then the frogs will go extinct. On the other hand, if the initial population is above ______ thousand frogs, then the frog population will eventually reach approximately ______ thousand.