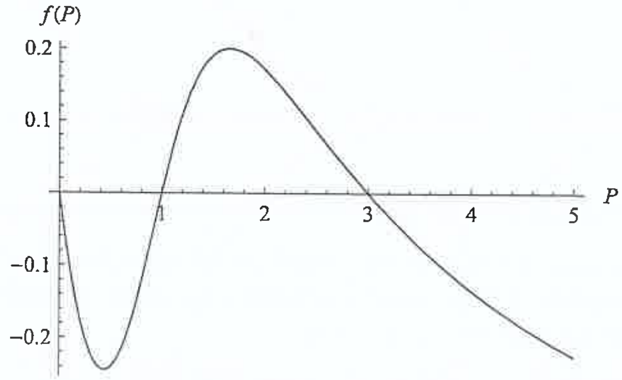
MAT 239 (Differential Equations), Prof. Swift
Worksheet 10 on Autonomous 1st Order ODEs

The population, $P(t)$, of green-eyed frogs on an island (measured in thousands of frogs) at time t (measured in months) obeys the differential equation

$$\frac{dP}{dt} = f(P),$$

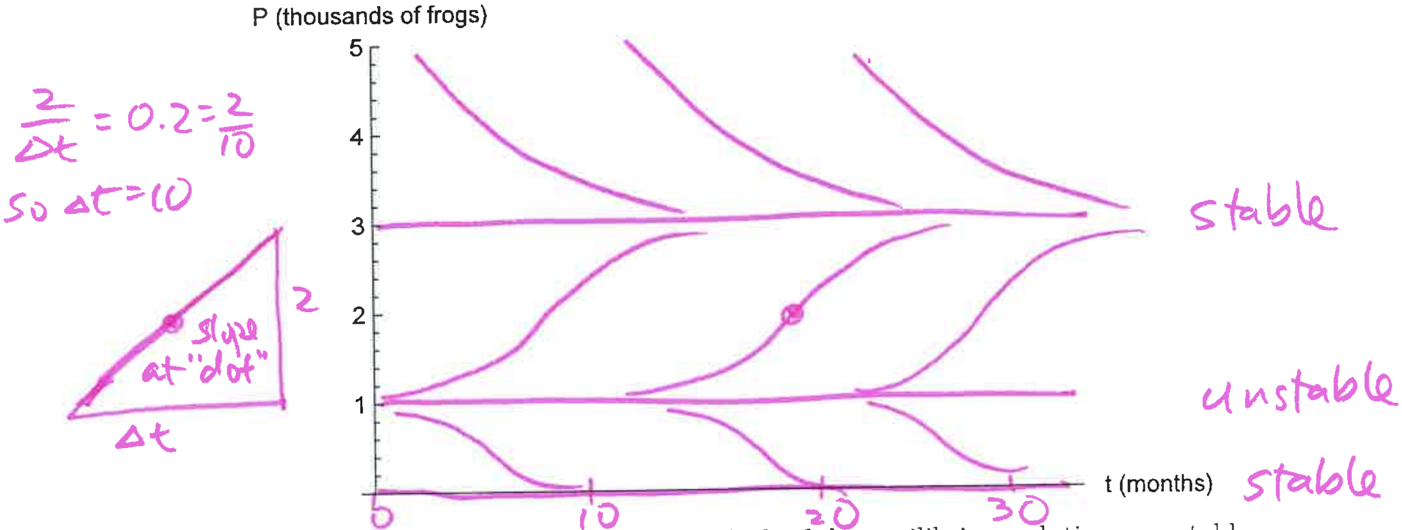
where the graph of f is shown below.



(a) Write down the equilibrium solutions (also called constant solutions) of this ODE.

P=0, P=1, P=3 (or P(t)=0, etc.)

(b) On the back of this page, sketch the graph of several solutions, $P(t)$, of this ODE. Be sure to include the equilibrium solutions. Put a few numbered tick marks on the t axis to indicate the approximate time scale. (Hint: The maximum value of $\frac{dP}{dt}$ is 0.2 thousand frogs per month.)



(c) Based on the graph of the solutions, which of the equilibrium solutions are stable, and which are unstable. (Hint: Nearby solutions approach a stable equilibrium and diverge away from an unstable equilibrium.)

$P=0$ is stable, $P=1$ is unstable, $P=3$ is stable

(d) Fill in the blanks. If the initial population of green-eyed frogs, $P(0)$, is below __1__ thousand, then the frogs will go extinct. On the other hand, if the initial population is above __1__ thousand frogs, then the frog population will eventually reach approximately __3__ thousand.