MAT 239 (Differential Equations), Prof. Swift
Worksheet 12 on Exact ODEs

1. Test if the following ODE's written in differential form are exact:

$3x^2ydx + x^3dy = 0$. Exact / Not Exact

$\frac{\partial}{\partial y}(3x^2y) \stackrel{?}{=} \frac{\partial}{\partial x}(x^3)$

$3x^2 = 3x^2$ ✓ Exact

$y^2dx + x^2dy = 0$. Exact / Not Exact

$\frac{\partial}{\partial y}(y^2) \stackrel{?}{=} \frac{\partial}{\partial x}(x^2)$

$2y \neq 2x$ NOT Exact

2. For each of the following, is the ODE equivalent to $\frac{dy}{dx} = \frac{y}{x}$? Then indicate if the original differential form of the ODE is exact.

$ydx - xdy = 0$. Equivalent / Not equivalent

$xdy = ydx$

$\frac{dy}{dx} = \frac{y}{x}$

Exact / Not Exact

$\frac{\partial}{\partial y}(y) \stackrel{?}{=} \frac{\partial}{\partial x}(-x)$

$1 \neq -1$ ∴ NOT exact

$\frac{1}{x}dx - \frac{1}{y}dy = 0$. Equivalent / Not equivalent

$\frac{1}{y}dy = \frac{1}{x}dx$

$dy = \frac{y}{x}dx$

$\frac{dy}{dx} = \frac{y}{x}$

Exact / Not Exact

$\frac{\partial}{\partial y}\left(\frac{1}{x}\right) \stackrel{?}{=} \frac{\partial}{\partial x}\left(\frac{-1}{y}\right)$

$0 = 0$

∴ exact

Moral: Same ODE can have 2 differential forms, one exact and the other not exact.