MAT 239 (Differential Equations), Prof. Swift
Worksheet 15, on finding the constants in "the" general solution

One version of WeBWorK problem 2 says "*The* general solution of a certain differential equation *can be written* as $y(t) = c_1e^{2t} + c_2e^{5t}$." Then they ask you to solve an IVP.

I have a different question for you (question 2) that will be super-important in this class. In problems 2 and 3 you get some practice solving for c_1 and c_2, and in problem 5 you learn the reason for the italics in the quote of WeBWorK problem 2.

1. What is that certain ODE from the WeBWorK problem?

2. Find the general solution to $y'' - y = 0$. Assume that t is the independent variable. You will use this general solution to solve 2 different IVPs in the next two problems.

3. Solve the IVP $y'' - y = 0, \quad y(0) = 1,\ y'(0) = 0$. Call this solution $y_a(t)$.

4. Solve the IVP $y'' - y = 0, \quad y(0) = 0,\ y'(0) = 1$. Call this solution $y_b(t)$.

5. The general solution to $y'' - y = 0$ can be written as $y(t) = ay_a(t) + by_b(t)$, where a and b are arbitrary constants. Find the constants c_1 and c_2 in terms of a and b, and then find a and b in terms of c_1 and c_2. Moral: *The* general solution *can be written* in many different ways.

6. Use the *new* form of the general solution to solve the IVP, $y'' - y = 0, \quad y(0) = 7,\ y'(0) = -3$.

Use what you know about y_a and y_b to find a: $y(0) = ay_a(0) + by_b(0) = \qquad\qquad = 7$

Use what you know about y_a and y_b to find b: $y'(0) = ay_a'(0) + by_b'(0) = \qquad\qquad = -3$

The solution, written in terms of $y_a(t)$ and $y_b(t)$, is