MAT 239 (Differential Equations), Prof. Swift
Worksheet 16, on the Most Beautiful Fundamental Solution Set.

The ODE $y''+p(x)y'+q(x)y=0$ can be written as $L[y]=0$, where $L=D^2+p(x)D+q(x)$. Assume that p and q are continuous at 0. We don't know p and q, so we have no hope of actually writing down the formula for any solutions other than $y(x)=0$.

Let $y_1(x)$ be the solution to the IVP $L[y]=0, \quad y(0)=1,\ y'(0)=0$. Write down what this tells you about the function $y_1(x)$. (Recall day one: What is a DE? What is a solution to a DE?)

Let $y_2(x)$ be the solution to the IVP $L[y]=0, \quad y(0)=0,\ y'(0)=1$. What do you know about y_2?

Use the properties of linear operators to show that $L[c_1y_1(t)+c_2y_2(t)]=0$ for all t.

You have just proved the superposition principle. If y_1 and y_2 are solutions to a linear homogeneous ODE, then $y=c_1y_1(t)+c_2y_2(t)$ is a solution to that same ODE for any real numbers c_1 and c_2. Find the solution to $L[y]=0, \quad y(0)=y_0,\ y'(0)=v_0$ for any y_0 and v_0. Since you have solved *any* IVP, you have shown that $y=c_1y_1(t)+c_2y_2(t)$ is the general solution to $L[y]=0$.

Let's put some flesh on those bones. The rest of the questions have one ODE with different ICs.

2. Solve the IVP $y''-4y=0, \quad y(0)=1, y'(0)=0$. Call this solution $y_1(x)$.

3. Solve the IVP $y''-4y=0, \quad y(0)=0, y'(0)=1$. Call this solution $y_2(x)$.

4. Without doing any more work, solve these IVPs:

Solve the IVP $y''-4y=0, \quad y(0)=3, y'(0)=2$.

Solve the IVP $y''-4y=0, \quad y(0)=-2, y'(0)=1$.

Solve the IVP $y''-4y=0, \quad y(0)=0, y'(0)=-2$.