MAT 239 (Differential Equations), Prof. Swift
Worksheet 17, on repeated and complex roots.

1. Find the general solution to $y'' - 6y' + 9y = 0$

$r^2 - 6r + 9 = 0$
$(r-3)^2 = 0$
$r_1 = r_2 = 3$

$y = C_1 e^{3t} + C_2 t e^{3t}$

or $y = (C_1 + C_2 t) e^{3t}$

2. Find the general solution to $y'' + 2y' + 5y = 0$

you can use quadratic formula, or complete the square, since this does not factor:

$r^2 + 2r + 5 = 0$
$r^2 + 2r = -5$
$r^2 + 2r + 1 = -5 + 1$
$(r+1)^2 = -4$
$r + 1 = \pm 2i$
$r = -1 \pm 2i$
$a = -1, \ b = 2$

$y = C_1 e^{-t} \cos(2t) + C_2 e^{-t} \sin(2t)$

or $y = e^{-t}(C_1 \cos(2t) + C_2 \sin(2t))$

3. Suppose one solution to a 2^{nd} order Linear Homogeneous ODE with Constant coefficients (LHODECC) is $y(t) = 5e^{2t}\cos(3t)$. What is the ODE? What is the general solution?

$r = a \pm bi$, where $a=2$, $b=3$. That is, $r = 2 \pm 3i$.

Do the completing the square backwards to get the characteristic equation:

$r = 2 \pm 3i$

$r - 2 = \pm 3i$

$(r-2)^2 = -9$

$r^2 - 4r + 4 = -9$

$r^2 - 4r + 13 = 0$

$\boxed{y'' - 4y' + 13y = 0}$ is the ODE

$y = C_1 e^{2t}\cos(3t) + C_2 e^{2t}\sin(3t)$ is the general solution. ($C_1 = 5$, and $C_2 = 0$ in the given solution.)