# MAT 239 (Differential Equations), Prof. Swift
## Worksheet 25, Power Series Review

A "nice" function is equal to its Taylor Series at all $x$ where the series converges. You should know this series, its interval of convergence, and its radius of convergence from Calc 2.

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots, \quad \text{for } -1 < x < 1, \ R = 1.$$

Using just this fact, write down the first four nonzero terms of the Taylor Series for these functions, and indicate their interval of convergence and radius of convergence. I gave a hint on the first one.

1. $f(x) = \frac{3}{1-2x} = 3\frac{1}{1-(2x)} = 3(1 + 2x + (2x)^2 + (2x)^3 + \ldots) = \boxed{3 + 6x + 12x^2 + 24x^3 + \ldots}$

2. $f(x) = \frac{x}{1+2x^2} = x \cdot \frac{1}{1-(-2x^2)} = x(1 + (-2x^2) + (-2x^2)^2 + (-2x^2)^3 + \ldots) = \boxed{x - 2x^3 + 4x^5 - 8x^7 + \ldots}$

3. In problem 2, you showed that $\frac{x}{1+2x^2} = \sum_{n=0}^{\infty} c_n x^n$, where

$c_0 =$ 0, $c_1 =$ 1, $c_2 =$ 0, $c_3 =$ −2, $c_4 =$ 0, $c_5 =$ 4, $c_6 =$ 0, $c_7 =$ −8.

no constant term

coefficient of $x^3$

4. Now, do your problem 5 on the WeBWorK.

1. converges if $|2x| < 1$ or $|x| < \frac{1}{2}$. The interval is $(-\frac{1}{2}, \frac{1}{2})$. The radius is $R = \frac{1}{2}$

2. Converges if $|-2x^2| < 1$ or $2x^2 < 1$ or $x^2 < \frac{1}{2}$

The Interval of convergence is $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$.
The Radius of Convergence is $R = \frac{1}{\sqrt{2}}$