

## MAT 239 (Differential Equations), Prof. Swift Worksheet 28, Eigenvalues and Eigenvectors

The eigenvalues/eigenvectors of a matrix  $A$  satisfy  $A\mathbf{v} = \lambda\mathbf{v}$ ,  $\mathbf{v} \neq \mathbf{0}$ , and  $\det(A - \lambda I) = 0$ .

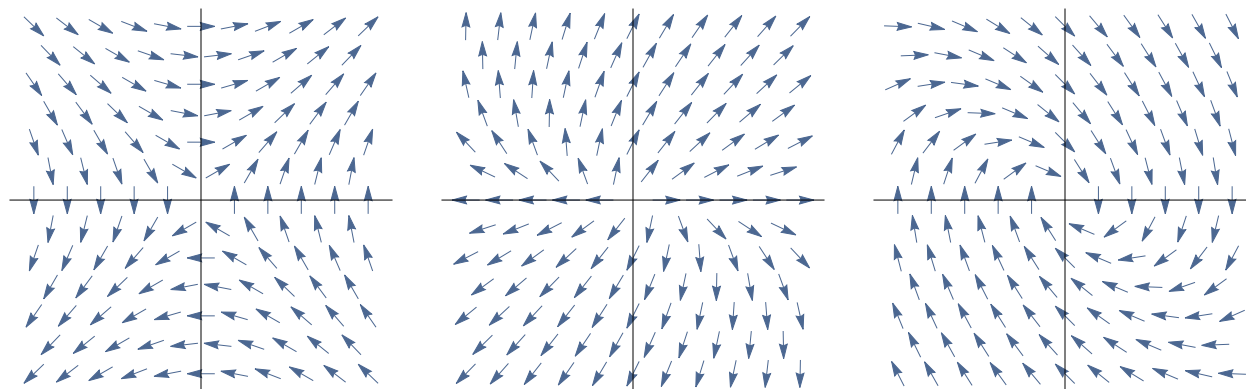
1. Let  $A = \begin{bmatrix} 4 & 3 & 1 \\ 1 & 5 & 1 \\ 0 & 1 & 9 \end{bmatrix}$  and  $\mathbf{v} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ .

Yes/No: Is  $A\mathbf{v} = 3\mathbf{v}$ ?

Yes/No: Does that imply that 3 is an eigenvalue of  $A$ ? Why not?

Yes/No: Is 3 an eigenvalue of  $A$ ? (Hint: The determinant of  $\begin{bmatrix} 1 & 3 & 1 \\ 1 & 2 & 1 \\ 0 & 1 & 6 \end{bmatrix}$  is  $-6$ .) Why not?

2. The figure shows 3 vector fields  $\mathbf{F}(\mathbf{x}) = A\mathbf{x}$ . If  $A$  has real eigenvalues, draw the lines that are the eigenvector directions, and indicate if the corresponding eigenvalue is positive or negative.



2. Compute the eigenvalues and eigenvectors of  $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ .