MAT 239 (Differential Equations), Prof. Swift
Eigenvalue hack, and solving $\mathbf{x}' = A\mathbf{x}$ when $\lambda_1 \neq \lambda_2$ are real

The eigenvalues λ_1 and λ_2 of a matrix 2×2 matrix A, satisfy

$$\lambda_1 + \lambda_2 = T \text{ and } \lambda_1 \cdot \lambda_2 = D$$

where $T = \mathrm{Tr}(A)$ is the trace of A (the sum of the diagonal entries) and $D = \mathrm{Det}(A)$ is the determinant of A. That is enough to find the eigenvalues of A in many cases. If that doesn't work, use the fact that the characteristic equation of A is $\lambda^2 - T\lambda + D = 0$.

1. Use this hack to find the eigenvalues of the following matrices:

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix} \qquad \begin{bmatrix} -3 & 4 \\ -2 & 1 \end{bmatrix}$$

2. Fill in the first row so the eigenvalues are 3 and -2.

$$\begin{bmatrix} & \\ 1 & 2 \end{bmatrix}$$

3. Find the general solution to the system $\dfrac{d\mathbf{x}}{dt} = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \mathbf{x}$, also written $\mathbf{x}' = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \mathbf{x}$.