

How to solve a 1st order Linear ODE

Step 1: Put ODE in standard form: $y' + p(t)y = g(t)$

Step 2: Compute "magic" integrating factor and simplify.

$$\mu(t) = \exp\left(\int p(t) dt\right)$$

Step 3: multiply both sides of standard form by $\mu(t)$.

$$\mu(t)y' + \mu(t)p(t)y = \mu(t)g(t)$$

Step 4: Rewrite the L.H.S. as $\frac{d}{dt}(\mu(t)y)$. Check using product rule

$$\frac{d}{dt}(\mu(t)y) = \mu(t)g(t)$$

Step 5: Integrate to get $\mu(t)y$, and solve for y .

General
Solution

$$y = \frac{\int \mu(t)g(t) dt + C}{\mu(t)}$$

Step 6: If I.C. is given, solve for C and substitute C into general solution.